# PAPERS READ

BEFORE

# The British Columbia Academy of Science

1910-1913

PRINTED BY
WESTERN SPECIALTY LIMITED
331 DUNSMUIR STREET, VANCOUVER, B.C
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# Inaugural Address

OF REV. G. W. TAYLOR, M.A., PRESIDENT.
(Read Dec. 3, 1910.)

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As this is really the beginning of our active work, I am not able tonight to speak of things accomplished. I may, however, remind you that the Academy has been founded under the most favorable circumstances. I am quite sure that the day is not distant when every scientific man and woman in the province will be not only willing but anxious to be admitted to membership in our society. Already applications are beginning to come in, with some of which we may deal tonight. That is all I need say of the past. Our Academy is now duly incorporated and fairly launched. It rests with us to justify our existence, and prove to the outside world that we are worthy of all the moral and material support the people of this province can give us.

I propose this evening (as material for a retrospect is lacking) to submit to you very briefly my own ideas of what the aim of the Academy should be, and to point out one or two ways in which we may all help to forward the objects for which our society has been established.

In a new country, science is almost the last subject to attract the attention of the masses of the people. The efforts of the first settlers are necessarily directed to the solution of the very pressing problem of how to win a livelihood on land or sea. From agriculture and hunting their attention may turn to trading or mining; but not perhaps for a long time will any attention at all be given to science or literature or art. This is of course quite natural; but after a time, surely, though all too slowly as many think, the claims of these things will be recognized.

I believe, gentlemen, that the time has arrived in British Columbia when the people, or at any rate those of the people who think, are prepared to admit that every industry owes much—owes almost everything in fact—to the discoveries of science; and they are prepared to welcome and to encourage scientific investigation on the part of those of their fellow countrymen who are willing to carry it on. Now that we have banded ourselves together for this very purpose, our primary aim must be and is to carry on through our members scientific research in every department possible.

Whether research is to lead to discoveries capable of immediate practical application or not, it is equally valuable; for all research must eventually result in practical benefit to mankind. Whether we are studying the material works of the Creator as biologists or geologists, or investigating the problems of abstract science, we are all helping to increase the round total of human knowledge; and that I believe is one of the highest and noblest objects which mortal man can strive to attain.

As regards the biological and geological sciences (I speak of them first, not because I claim them as of first importance, but because they have been the subjects of my own studies), in British Columbia we have a most fertile and comparatively little-worked field. I am not ignorant of what has been accomplished in this field by eminent foreigners and by our fellow-Canadians from the eastern provinces. But the results of their investigations have been largely recorded in publications not accessible to the general public in this country. They are buried in the transactions of learned bodies in other lands, and are not therefore of as much service as they might be to us here. Our endeavor must be to carry their investigations farther, and to secure the publication of our results in British Columbia, in such a form that they will be intelligible as well as accessible to the man in the street. To study at home, and when we have studied to publish at home the fruits of that study,—that to my mind must be our first endeavor. This means not only work on our part, but also the expenditure of money. It will be a long time, I fear, before our annual subscriptions will suffice to pay our printers' bill. We shall be obliged to start a special publication fund. We shall have to make sacrifices ourselves in order to secure the prompt printing of the papers presented. But, gentlemen, let us begin with the resolve that no matter what sacrifices on our part are involved, every paper that is presented to this Academy which is worth printing shall be printed.

The Dominion, and some of the provincial, governments give very substantial grants every year to various societies to help them in the publication of their proceedings. Indeed, the government of Quebec went so far as to make a grant of $500 a year to an individual, to aid him in carrying on the publication of a scientific monthly in that province.

Up to the present there have been many difficulties to be faced by men endeavoring to carry on original research in this province. Isolation from fellow-workers has been a great disadvantage. This we shall remove as far as is at present possible by bringing together in this Academy all scientific workers. Here at any rate we shall find some of the sympathy we have lacked so long.

But a still more serious drawback, and one not so easy to overcome, has been the absence of libraries, and the consequent difficulty—the absolute impossibility I might almost call it—of keeping abreast of the times. This drawback will be gradually lessened in some cases. There will in time be a good biological library at the station in Nanaimo. When our provincial university is founded, there will doubtless be a good collection of books in many departments of science. But we must have a library of our own as soon as possible, for we must endeavor to help our workers with tools. An adequate library for reference must be our ultimate aim, but we cannot attain this all at once. We can, however, make a beginning. It is a matter in which all can help. Each member can present the Academy with at least one book,—a duplicate maybe from his own library, or a book dealing with some branch outside his own special interests.

Just as soon, moreover, as we begin our own publication, we shall be able to add to our library by exchanging with other scientific societies. The Royal Society of Canada, the Canadian Institute, the Geological Survey of Canada, the government bureaus of the United States, and various other bodies have either already promised or may be confidently expected to co-operate with us in this way.

I may remind you that we are already on the Royal Society's list of associated societies; and as I was present at its last annual meeting in May, I can tell you that great was the interest shown by the Fellows in the effort we are making, and many were the kind words and good wishes in reference to what they termed "the youngest scientific society in the Dominion."

I think then that we should take steps immediately to begin the formation of a library, trusting that in the ways I have indicated, and perhaps through the generosity of our members and others, it will soon assume useful proportions.

I have said that we must justify our existence. We shall have no difficulty in doing that. We must, however, go further and secure popular sympathy, for we shall not then lack popular support. With a field such as we possess, with almost limitless possibilities in every department of science, the extent and value of our work will depend entirely upon the ability and energy of our workers. Popular sympathy, you know, is always on the side of the workers.

We must realize then that the success of the Academy will not depend upon the Biological Section, or the Physical Section, or the Mathematical Section, but upon the *working* section. Each member must belong

to that section, and must make up his mind to contribute his quota—large or small, according to his opportunities—to the proceedings of the Academy. By these proceedings will the Academy be judged. If you all do your part, success is assured; but even if you don't—even if we are limited to a bare half-dozen workers, and have to place all our other members in an ornamental section, we may still succeed. For half-a-dozen men, or even a less number than that, filled with a spirit of enthusiasm and optimism (the two go together) can overcome any difficulty, and make any movement a success. A lack of such a spirit has brought failure to many an ambitious scheme, but I know that there will be no such lack here. For I can count among our present members more than the half-dozen of men whom I know will not fail us; men who have in them that keen desire to fathom the unknown—to understand the works of the Creator—to unravel the secrets of Nature—which is never satisfied, but is ever spurring on to fresh effort. And these men will make our Academy a success. For my own part I shall try to do my share.

(NOTE.—This inaugural address must be taken as the parting word of our first president. He died at Nanaimo on August 22nd, 1912. An appreciation of his work appeared in the "Proceedings of the B. C. Entomological Society," 1912, and a very full biographical article with a good portrait, was published in the "Transactions of the Royal Society of Canada" for 1913.—J.P.)

# A Geometrical Vector Algebra

By T. PROCTOR HALL, M.A., Ph. D., M.D.

1. The laws of operation of any algebra are ultimately based upon its definitions. If the definitions are geometrical the algebraic operations have geometric correspondences. The operations of addition and subtraction in common algebra, for example, correspond to the geometric addition and subtraction of straight lines, vectors, surfaces, etc.

In this algebra new definitions of vector multiplication and division are adopted, in consequence of which all algebraic operations upon vectors (directed unlocated straight lines or steps), or rather upon vector symbols, correspond to geometric operations in space upon the vectors themselves; and every algebraic vector expression corresponds to some geometric configuration of the vectors themselves.

In every vector demonstration or problem, therefore, the student may think in terms of either algebra or geometry or both; and may at any time change from one realm of thot to the other with no break in the continuity.

This algebra is developed first in terms of analytical geometry for three-fold space, and is then adapted to two-fold and to four-fold space. Complex numbers, spherical trigonometry, and quaternion rotations, appear as special cases.

2. NOTATION.—Taking three rectangular axes X, Y, Z, let $\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}$ denote unit vectors (steps) outward from the centre O, along the axes. Unit vectors in the opposite direction from O are denoted by $\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{z}}$. Vectors in general are herein denoted by black faced Gothic capitals, and the corresponding unit vectors by black faced italics. For purposes of designation and operation all vectors (unless otherwise indicated) are understood to start from O, the centre of coordinates.

Then if $\mathbf{A}$ is any vector, $a$ is its length, $\boldsymbol{a}$ is unit length of the same vector, $a_x \boldsymbol{x}$, $a_y \boldsymbol{y}$, $a_z \boldsymbol{z}$ are the vector components of $\mathbf{A}$ along X, Y, Z, and $a_x, a_y, a_z$ are the lengths of these components.

Then $\mathbf{A} = a\boldsymbol{a}$

$= a_x \boldsymbol{x} + a_y \boldsymbol{y} + a_z \boldsymbol{z}$ by vector addition.

$a^2 = a_x^2 + a_y^2 + a_z^2$ by solid geometry.

The symbol $\mathbf{A}$ is used to indicate (1) the vector from O to the point whose rectangular coordinates are $a_x, a_y, a_z$; (2) motion from O to the extremity of $\mathbf{A}$; (3) a rotor, defined in §7.

The line or locus of $\mathbf{A}$ is expressed by an elongated $l$, thus $l\mathbf{A}$, and any part of this locus, from $m$ to $n$, is written $_{m}l\,\mathbf{A}$. Surface loci are ordinarily expressed by two $l$'s and solid loci by three $l$'s.

**3.** To express the cosine of the angle between two vectors in terms of the coordinates of the vectors.

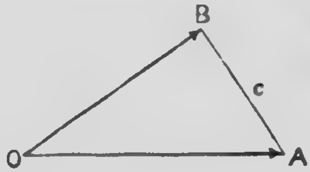


Fig. 1.

Let $c$ be the length of the line joining the extremities of the vectors $\mathbf{A}$, $\mathbf{B}$, from O.

By solid geometry—

$$c^2 = (a_x - b_x)^2 + (a_y - b_y)^2 + (a_z - b_z)^2$$

By plane trigonometry—

$$c^2 = a^2 + b^2 - 2ab\cos \mathbf{A}\,\mathbf{B}.$$

$$\text{Therefore, } \cos \mathbf{A}\,\mathbf{B} = \frac{a_x\, b_x + a_y\, b_y + a_z\, b_z}{a\, b} \equiv \frac{S_{ab}}{a\, b},$$

where $S_{ab} \equiv$ the sum of the $a\,b$ products $= a\,b \cos \mathbf{A}\,\mathbf{B}$.

If $S_{ab} = 0$, $\mathbf{A} \perp \mathbf{B}$, and conversely.

Example 1.—Find the angle between the vectors $\mathbf{x} + 2\mathbf{y}$ and $2\mathbf{x} - \mathbf{y} + c\mathbf{z}$. Here $S = 0$, and the vectors are perpendicular.

Example 2.—What angles does the vector $2\mathbf{x} - \mathbf{y} + \mathbf{z}$ $(= \mathbf{A})$ make with the axes $\mathbf{x}$, $\mathbf{y}$, $\mathbf{z}$?

$$a = \sqrt{6};\ \therefore \cos \mathbf{A}\,\mathbf{x} = \frac{2}{\sqrt{6}},\quad \cos \mathbf{A}\,\mathbf{y} = \frac{-1}{\sqrt{6}},\quad \cos \mathbf{A}\,\mathbf{z} = \frac{1}{\sqrt{6}}.$$

**4.** ADDITION AND SUBTRACTION.—Addition is geometrically defined as the process of making the second vector step from the extremity of the first. The sum is the new vector from O to the extremity of the second vector thus added.

Algebraically addition is performed by resolving the vectors into their components and adding these.

$$\begin{aligned}\mathbf{A} + \mathbf{B} &= (a_x\,\mathbf{x} + a_y\,\mathbf{y} + a_z\,\mathbf{z}) + (b_x\,\mathbf{x} + b_y\,\mathbf{y} + b_z\mathbf{z}) \\ &= (a_x + b_x)\,\mathbf{x} + (a_y + b_y)\,\mathbf{y} + (a_z + b_z)\,\mathbf{z}.\end{aligned}$$

Subtraction is addition of the negative of a vector.

Hence, both geometrically and algebraically vector terms are commutative.

$$\mathbf{A} \pm \mathbf{B} = \pm \mathbf{B} + \mathbf{A}.$$

**5.** COLLINEAR VECTORS.—Two vectors $\mathbf{A}$, $\mathbf{B}$, are in the same line when

$$\mathbf{A} \equiv n\mathbf{B},$$

or

$$\frac{a_x}{b_x} = \frac{a_y}{b_y} = \frac{a_z}{b_z} = n.$$

If $n$ is positive, $\mathbf{A}$ and $\mathbf{B}$ are in the same direction; if negative, $\mathbf{A}$ and $\mathbf{B}$ are opposite. If $n = 1$, $\mathbf{A} \equiv \mathbf{B}$.

**6.** COPLANAR VECTORS.—Three vectors, $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, are in the same plane when another vector, $\mathbf{K}$, can be found which is perpendicular to each of them. Then $S_{ak} = S_{bk} = S_{ck} = 0$, by §3. Eliminating $k_x$, $k_y$, $k_z$ we get the coplanar equation

$$| \, a_x \, b_y \, c_z \, | = 0.$$

The determinant $| \, a_x \, b_y \, c_z \, |$ is six times the volume of the tetrahedron whose corners are O A B C. When this volume is zero $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, are coplanar.

EXAMPLE.—Find the conditions under which $\mathbf{A}$ is perpendicular to $\mathbf{C} = \boldsymbol{x} + \boldsymbol{z}$, and in the $\mathbf{B}$ $\mathbf{C}$ plane where $\mathbf{B} = 2\boldsymbol{x} - \boldsymbol{y}\sqrt{3}$.

The condition of perpendicularity is

$$S_{ac} = 0, \qquad \text{or } a_x + a_z = 0.$$

The coplanar equation is

$$\begin{vmatrix} 2 & -\sqrt{3} & 0 \\ 1 & 0 & 1 \\ a_x & a_y & a_z \end{vmatrix} = 0.$$

Therefore $\mathbf{A} = a_x \, (\boldsymbol{x} - \boldsymbol{y}\sqrt{3} - \boldsymbol{z})$.

**7.** MULTIPLICATION of the vector $\mathbf{B}$ by the vector $\mathbf{A}$ is written $\mathbf{A}\,\mathbf{B}$, and is defined geometrically as the combined operations,

(1) Extension of $\mathbf{B}$ until its length is $ab$,

(2) Simultaneous rotation of $\mathbf{B}$ thru 90° about $\mathbf{A}$ as an axis, in a direction which is right handed or clockwise when facing in the positive direction of $\mathbf{A}$.

Each vector multiplier is a tensor-rotor. The rotor power of all vectors is the same and needs no separate expression at this stage.

The product $\mathbf{AB}$ is that vector from O whose extremity is the final position of the point $\mathbf{B}$ after extension and rotation. The locus of $\mathbf{AB}$ is the curve traced by the point $\mathbf{B}$ during the operation.

$\mathbf{ABC}$ means the operation of $\mathbf{A}$ on the product $\mathbf{BC}$, or

$$\mathbf{ABC} \equiv \mathbf{A}.\,\mathbf{BC}.$$

$$\text{Also} \qquad \mathbf{A}^2\,\mathbf{B} \equiv \mathbf{A}.\,\mathbf{AB}, \text{ etc.}$$

It next becomes necessary to find the laws of algebraic multiplication that correspond to the geometric changes here defined.

**8.** Multiplication by a collinear vector makes no change except in length or sign,

$$\mathbf{x}\mathbf{x} = \mathbf{x}$$
$$\mathbf{x}\bar{\mathbf{x}} = \bar{\mathbf{x}}$$
$$\bar{\mathbf{x}}\mathbf{x} = \bar{\mathbf{x}}$$
$$\bar{\mathbf{x}}\bar{\mathbf{x}} = \mathbf{x}$$
$$\mathbf{A}\boldsymbol{a} = \mathbf{A}, \text{ etc.}$$

**9.** Unit perpendicular vectors give the following results which are geometrically evident.

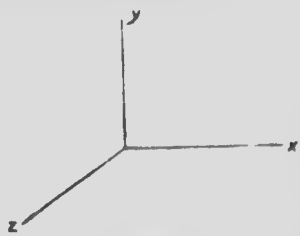


FIG. 2

$$\mathbf{x}\mathbf{y} = \mathbf{z} \qquad \bar{\mathbf{x}}\mathbf{y} = \bar{\mathbf{z}}$$
$$\mathbf{x}\mathbf{z} = \bar{\mathbf{y}} \qquad \bar{\mathbf{x}}\bar{\mathbf{z}} = \bar{\mathbf{y}}$$
$$\mathbf{x}\bar{\mathbf{y}} = \bar{\mathbf{z}} \qquad \bar{\mathbf{x}}\bar{\mathbf{y}} = \mathbf{z}$$
$$\mathbf{x}\bar{\mathbf{z}} = \mathbf{y} \qquad \bar{\mathbf{x}}\mathbf{z} = \mathbf{y}$$

and similarly for $\mathbf{y}$ and $\mathbf{z}$ as operators. Here the laws of signs are the same as in common algebra, so long as the factors are in alphabetical circular order;

$$\bar{\mathbf{x}}\bar{\mathbf{y}} = \mathbf{z} = \mathbf{x}\mathbf{y},$$
$$\mathbf{x}\bar{\mathbf{y}} = \bar{\mathbf{z}} = \bar{\mathbf{x}}\mathbf{y} = -\,\mathbf{x}\mathbf{y}.$$

But reversing the order of the factors changes the sign of the product;

$$\mathbf{x}\mathbf{y} = \mathbf{z}$$
$$\mathbf{y}\mathbf{x} = \bar{\mathbf{z}}.$$

The second power of an unit perpendicular operator is equivalent to -1,

$$\mathbf{x}^2\mathbf{y} = \mathbf{x}\mathbf{z} = \mathbf{y}$$
$$\bar{\mathbf{x}}^2\mathbf{y} = \bar{\mathbf{x}}\bar{\mathbf{z}} = \bar{\mathbf{y}}.$$

The fourth power leaves the operand unchanged,

$$\mathbf{x}^4\mathbf{y} = \mathbf{x}^2\bar{\mathbf{y}} = \mathbf{y}.$$

When the vectors are not units the product of their tensors is prefixed to the vector product,

$$a\mathbf{x}.\ b\mathbf{y} = ab.\ \mathbf{x}\mathbf{y} = ab\mathbf{z}.$$

**10.** To find the algebraic product of any two vectors. Let the product be $\mathbf{K} = \mathbf{AB}$. Draw KD ⊥ O A, and OV equal and parallel to D K. Then the length OD is

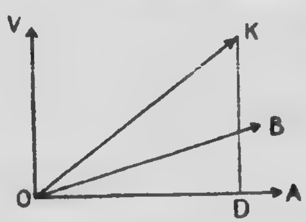


FIG. 3

$$\text{OD} = \text{OK} \cos \mathbf{AK}$$
$$= ab \cos \mathbf{AB}$$
$$= \mathrm{S}_{ab}, \qquad by\ \S 8.$$

As vectors $\text{OK} = \text{OD} + \text{DK}$,

or $\mathbf{K} = \mathrm{S}_{ab}\,\boldsymbol{a} + \mathbf{V}$.

To determine **V** we have the equations of perpendicularity,

$$S_{av} = a_x v_x + a_y v_y + a_z v_z = o$$
$$S_{bv} = b_x v_x + b_y v_y + b_z v_z = o,$$

and from the triangle ODK,

$$v_x^2 + v_y^2 + v_z^2 = v^2 = a^2 b^2 - S^2.$$

Solving we get

$$v_x = a_y b_z - a_z b_y$$
$$v_y = a_z b_x - a_x b_z$$
$$v_z = a_x b_y - a_y b_x.$$

Hence the "Vector Normal" to **A**, **B**, is

$$\mathbf{V} = \begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ \boldsymbol{x} & \boldsymbol{y} & \boldsymbol{z} \end{vmatrix}$$

and its length is

$$v = \sqrt{a^2 b^2 - S^2} = ab \sin \mathbf{AB}.$$

The product **K** is thus expressed in terms of the given vectors and their components, in the equation

$$\mathbf{AB} = S\boldsymbol{a} + \mathbf{V}.$$

Example. Find the product **AB** when

$$\mathbf{A} = 3\boldsymbol{y} - \boldsymbol{z}, \quad \mathbf{B} = 3\boldsymbol{z} - \boldsymbol{y}.$$

Here $\quad S = -6, \quad \mathbf{V} = 8\boldsymbol{x}, \quad a = \sqrt{10},$

$$\therefore \mathbf{AB} = 8\boldsymbol{x} - \frac{3\sqrt{10}}{5}(3\boldsymbol{y} - \boldsymbol{z}).$$

**11.** PERMUTATION OF FACTORS. It is geometrically and algebraically evident that

$$S_{ab} = S_{ba}$$

and that $\quad \mathbf{V}_{ab} = -\mathbf{V}_{ba}.$

Hence
$$\mathbf{BA} = S_{ba}\boldsymbol{b} + \mathbf{V}_{ba}$$
$$= S_{ab}\boldsymbol{b} - \mathbf{V}_{ab}$$

which is not equal to **AB**.

Changing the order of the factors changes the vector product. Vectors are not permutable.

**12.** OPERAND DISTRIBUTIVE. To find the product $\mathbf{A}(\mathbf{B} \pm \mathbf{C})$

let $\quad \mathbf{B} \pm \mathbf{C} = \mathbf{D},$

so that
$$d_x = b_x \pm c_x$$
$$d_y = b_y \pm c_y$$
$$d_z = b_z \pm c_z.$$

Then
$$\begin{aligned}\mathbf{A}\,(\mathbf{B} \pm \mathbf{C}) &= \mathbf{AD} \\ &= \mathrm{S}_{ad}\,\boldsymbol{a} + \mathbf{V}_{ad} \\ &= (\mathrm{S}_{ab} \pm \mathrm{S}_{ac})\,\boldsymbol{a} + \mathbf{V}_{ab} \pm \mathbf{V}_{ac} \\ &= (\mathrm{S}_{ab}\,\boldsymbol{a} + \mathbf{V}_{ab}) \pm (\mathrm{S}_{ac}\,\boldsymbol{a} + \mathbf{V}_{ac}) \\ &= \mathbf{AB} \pm \mathbf{AC}.\end{aligned}$$

The operand is therefore distributive.

13. OPERATOR NOT DISTRIBUTIVE. To find the product $(\mathbf{A} \pm \mathbf{B})\mathbf{C}$, let $\mathbf{A} \pm \mathbf{B} = \mathbf{K}$ so that
$$\begin{aligned}k_x &= a_x \pm b_x \\ k_y &= a_y \pm b_y \\ k_z &= a_z \pm b_z\end{aligned}$$
and
$$k^2 = a^2 + b^2 \pm 2\,\mathrm{S}_{ab}, \text{ by § 3.}$$

Then
$$\begin{aligned}(\mathbf{A} \pm \mathbf{B})\mathbf{C} &= \mathbf{K}\mathbf{C} \\ &= \mathrm{S}_{kc}\,\boldsymbol{k} + \mathbf{V}_{kc} \\ &= \frac{\mathrm{S}_{ac} \pm \mathrm{S}_{bc}}{\sqrt{a^2 + b^2 \pm 2\,\mathrm{S}_{ab}}}\,(\mathbf{A} \pm \mathbf{B}) + (\mathbf{V}_{ac} \pm \mathbf{V}_{bc})\end{aligned}$$

which is not equal to $\mathbf{AC} \pm \mathbf{BC}$.

Hence the operator is not in general distributive.

14. FACTORS MUST NOT CHANGE ASSOCIATION.
$$\begin{aligned}\mathbf{ABC} &= \mathbf{A}\,(\mathrm{S}_{bc}\,\boldsymbol{b} + \mathbf{V}_{bc}) \\ &= \frac{\mathrm{S}_{bc}}{b}\,\mathbf{AB} + \mathbf{A}\mathbf{V}_{bc} \\ &= \frac{\mathrm{S}_{ab}\,\mathrm{S}_{bc}}{a\,b}\,\mathbf{A} + \frac{\mathrm{S}_{bc}}{b}\,\mathbf{V}_{ab} + |\,a_x\,b_y\,c_z\,|\,\boldsymbol{a} + \mathrm{S}_{ac}\,\mathbf{B} - \mathrm{S}_{ab}\,\mathbf{C}.\end{aligned}$$

To expand $\mathbf{AB}.\,\mathbf{C}$, let $\mathbf{K} = \mathbf{AB}$, so that
$$\mathbf{AB}.\,\mathbf{C} = \mathbf{KC} = \mathrm{S}_{kc}\,\boldsymbol{k} + \mathbf{V}_{kc}$$
$$\left\{\frac{\mathrm{S}_{ab}\,\mathrm{S}_{ac}}{a} + |\,a_x\,b_y\,c_z\,|\right\}\boldsymbol{a}\,\boldsymbol{b} + \frac{\mathrm{S}_{ab}}{a}\,\mathbf{V}_{ac}$$
$$+ \begin{vmatrix} |\,a_y\,b_z\,|, & |\,a_z\,b_x\,|, & |\,a_x\,b_y\,| \\ c_x & c_y & c_z \\ \boldsymbol{x} & \boldsymbol{y} & \boldsymbol{z} \end{vmatrix}$$

which is not equal to $\mathbf{ABC}$.

Hence the association of a factor must not in general be altered. But if $\mathbf{C} = \mathbf{A}$ these two products become identical, and therefore
$$\mathbf{A}.\,\mathbf{BA} = \mathbf{AB}.\,\mathbf{A}.$$

**15.** POWERS OF AN OPERATOR.

$$\mathbf{AB} = S\boldsymbol{a} + \mathbf{V}$$

$$\begin{aligned} \mathbf{A}^2\mathbf{B} &= \mathbf{A}.\,\mathbf{AB} = \mathbf{A}\,(S\boldsymbol{a} + \mathbf{V}) \\ &= S\mathbf{A} + \mathbf{AV} \\ &= 2\,S\mathbf{A} - a^2\mathbf{B} \text{ by expansion and multiplication.} \end{aligned}$$

$$\begin{aligned} \mathbf{A}^3\mathbf{B} &= \mathbf{A}\,(2\,S\mathbf{A} - a^2\mathbf{B}) \\ &= 2\,a^2\,S\boldsymbol{a} - a^2\,(S\boldsymbol{a} + \mathbf{V}) \\ &= a^2\,(S\boldsymbol{a} - \mathbf{V}), \text{ which is geometrically evident.} \end{aligned}$$

$$\begin{aligned} \mathbf{A}^4\mathbf{B} &= a^2\,S\boldsymbol{a} - a^2\,(S\boldsymbol{a} - a^2\mathbf{B}) \\ &= a^4\,\mathbf{B}, \text{ which is also geometrically evident.} \end{aligned}$$

From these results it is easy to write the expansion of any value of $\mathbf{A}^n\mathbf{B}$ when $n$ is a positive integer.

**16.** LAWS OF MULTIPLICATION, summary.

(1) Factors are not permutable (§11)

$\mathbf{AB}$ is not equal to $\mathbf{BA}$.

(2) The operand is distributive (§12)

$\mathbf{A}\,(\mathbf{B} \pm \mathbf{C}) = \mathbf{AB} \pm \mathbf{AC}$,

but the operator is not (§13)

$(\mathbf{A} \pm \mathbf{B})\,\mathbf{C}$ is not equal to $\mathbf{AC} \pm \mathbf{AC}$.

(3) The association of a factor must not be changed (§14).

$\mathbf{A}.\,\mathbf{BC}$ is not equal to $\mathbf{AB}.\,\mathbf{C}$

but $\mathbf{A}.\,\mathbf{BA} = \mathbf{AB}.\mathbf{A}$.

(4) The fourth power of an operator is equivalent to the fourth power of its tensor (§15).

(5) The common laws of signs are true for operand and product; not for the operator.

**17.** PERPENDICULAR VECTORS. When $\mathbf{A}, \mathbf{B}, \mathbf{C}$ are perpendicular, $S_{ab} = S_{ac} = S_{bc} = o$, (§3), and $\mathbf{AB} = \mathbf{V}$, (§15).

Then the laws of §16 become the following:

(1) Permuting the factors, i.e., interchanging operator and operand, changes the sign of the product.

$\mathbf{BA} = \mathbf{V}_{ba} = -\,\mathbf{V}_{ab} = -\,\mathbf{AB}$.

(2) Both operator and operand are distributive.

$$(\mathbf{A} \pm \mathbf{B})\mathbf{C} = \mathbf{AC} \pm \mathbf{BC}$$
$$\mathbf{A}\,(\mathbf{B} \pm \mathbf{C}) = \mathbf{AB} \pm \mathbf{AC}.$$

(3) The association of a factor must not be changed.

$\mathbf{A}.\,\mathbf{BC}$ is not equal to $\mathbf{AB}.\mathbf{C}$.

but $\mathbf{A}.\,\mathbf{BA} = \mathbf{AB}.\,\mathbf{A}$.

(4) The square of an operator is -1 times the square of its tensor.

$$\mathbf{A}^2\mathbf{B} = -a^2\mathbf{B}.$$

(5) The common laws of signs hold true.

If $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$ be any three unit perpendicular vectors in the same circular order as $\boldsymbol{x}$, $\boldsymbol{y}$, $\boldsymbol{z}$; then $\boldsymbol{ab} = \boldsymbol{c}$, $\boldsymbol{bc} = \boldsymbol{a}$, $\boldsymbol{ca} = \boldsymbol{b}$, and these vectors may serve as units of the system, as well as $\boldsymbol{x}$, $\boldsymbol{y}$ and $\boldsymbol{z}$.

18. DIVISION is the inverse of multiplication, so that if

$$\mathbf{AB} = \mathbf{C}$$

$$\frac{\mathbf{C}}{\mathbf{A}} = \mathbf{A}^{-1}\mathbf{C} = \mathbf{B}.$$

Geometrically, division is a negative turn of 90° about the divisor (identical in this respect with multiplication by the negative of the divisor) and reduction in length to that given by the quotient of the tensors.

19. GENERAL FORMULA for $\mathbf{A}^n\mathbf{B}$ where $n$ is real.

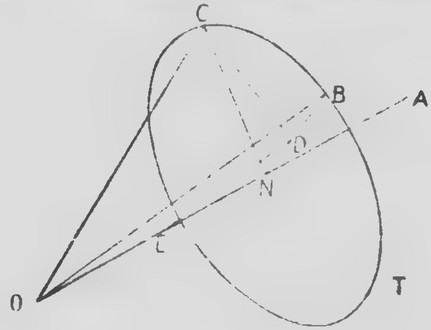



Fig. 4

Let O A, O B, be the vectors $\mathbf{A}$, $\mathbf{B}$, and let O C be in line with their vector product, so that $\mathbf{A}^n\mathbf{B} = a^n$ times O C.

Let B C T be the circle of revolution of $\mathbf{B}$ about $\mathbf{A}$; N its centre; N B, N C, its radii. Draw C D $\perp$ N B; D E $\parallel$ B O. Let $\angle$ C N B $= \theta = n\frac{\pi}{2}$ be the angle of rotation of $\mathbf{B}$.

Then $NC = NB = \dfrac{v}{a}$, where $v$ is the length of $\mathbf{V}_{ab}$.

$$DC = \frac{v}{a}\ \sin\ \theta,$$

$$DE = BO\ \frac{ND}{NB} = b\ \cos\ \theta,$$

$$OE = DB\ \frac{ON}{NB} = \frac{v}{a}\ vers\ \theta.\ cot\ \mathbf{AB}$$

$$= \frac{v}{a}\ vers\ \theta.\ \frac{s}{v} = \frac{s}{a}\ vers\ \theta.$$

As vectors $\quad OC = OE + ED + DC$

$$= \frac{S}{a}\,\boldsymbol{\alpha}\ vers\,\theta + \mathbf{B}\cos\theta + \frac{\mathbf{V}}{a}\sin\theta$$

$$\therefore\ \mathbf{A}^n\mathbf{B} = a^n.\ OC$$

$$= a^{n-1}\left(S\boldsymbol{\alpha}\ vers\,\theta + a\,\mathbf{B}\cos\theta + \mathbf{V}\sin\theta\right).$$

This formula, being true for all real values of $n$, includes products, quotients, powers and roots of vector operators.

Example.—Two rods, A and B, are joined at one end. **A** is one foot long, and the perpendicular distance of its free end from B is six inches. B is turned 60° about the axis of A, then **A** is turned 90° in the same direction about the new axis of B. Find the new position of **A**.

Let the joined ends be at O. Let $\mathbf{B} = b\boldsymbol{x}$, and $\mathbf{A} = a_x\boldsymbol{x} + a_y\boldsymbol{y}$. Since $a = 1$, and $a_y = \frac{1}{2}$, $\quad \mathbf{A} = \frac{1}{2}(\boldsymbol{x}\sqrt{3} + \boldsymbol{y})$.

The result of the first rotation is represented by

$$\mathbf{C} = \mathbf{A}^{\frac{2}{3}}\mathbf{B} = S\boldsymbol{\alpha}\ vers\,60^\circ + \mathbf{B}\cos 60^\circ + \mathbf{V}\sin 60^\circ$$

$$= \frac{b}{8}\left(7\boldsymbol{x} + \boldsymbol{y}\sqrt{3} - 2\boldsymbol{z}\sqrt{3}\right).$$

The second rotation is

$$\mathbf{c}\mathbf{A} = S\mathbf{c} + \mathbf{V}_{ca}$$

$$= \frac{1}{16}\left(9\boldsymbol{x}\sqrt{3} - 3\boldsymbol{y} - 2\boldsymbol{z}\right),$$

which gives the final position of the free end of **A**.

## 20. QUATERNIONS.

When $\quad \mathbf{A} \perp \mathbf{B},\ S = 0$

and $\qquad \mathbf{V} = \mathbf{AB}. \qquad$ (§17).

Then $\qquad \mathbf{A}^n\mathbf{B} = a^n\left(\mathbf{B}\cos\theta + \boldsymbol{\alpha}\mathbf{B}\sin\theta\right)$

$$= a^n\left(\cos\theta + \boldsymbol{\alpha}\sin\theta\right)\mathbf{B}.$$

Now $\boldsymbol{\alpha}^2$ as a perpendicular operator is equivalent to -1; and by expansion in series, exactly as with the complex $(\cos\theta + i\sin\theta)$, it may be shown that the rotor of $\mathbf{A}^n$

$$\cos\theta + \boldsymbol{\alpha}\sin\theta = e^{\boldsymbol{\alpha}\theta}$$

where $\theta$ is the angle and $\boldsymbol{\alpha}$ the axis of rotation.

Hence for perpendicular vectors

$$\mathbf{A}^n\mathbf{B} = a^n e^{\boldsymbol{\alpha}\theta}\,\mathbf{B}.$$

The operator $\mathbf{A}^n$ is a tensor-rotor-vector, or a directed quaternion, when applied to vectors perpendicular to $\mathbf{A}$. It has the four fundamental characters of a quaternion, namely,

(1) Since $\mathbf{A}^n\ \mathbf{B} = \mathbf{C}$, $\mathbf{A}^n$ may be regarded as the ratio of $\mathbf{C}$ to $\mathbf{B}$;

(2) It is the product of a tensor and a directed rotor, $a^n$. $e^{\mathbf{a}\theta}$;

(3) It is the sum of a scalar or number and a directed unlocated line or vector, $a^n \cos\theta + a^n \mathbf{a} \sin\theta$;

(4) It is a quadrinomial of the form $k + l\mathbf{x} + m\mathbf{y} + n\mathbf{z}$, where $k$ is a pure number and the directive units $\mathbf{x}$, $\mathbf{y}$, $\mathbf{z}$, have the relations
$\mathbf{x}^2 = \mathbf{y}^2 = \mathbf{z}^2 = \mathbf{xyz} = -1.$

**21. VECTOR ARCS.** The rotor $e^{\mathbf{a}}$ turns thru the angle $a$ about the axis $\mathbf{A}$ any vector in the plane perpendicular to $\mathbf{A}$. The index $\mathbf{a}$ is a vector angle whose axis is $\mathbf{A}$ and whose magnitude is $a$ radians. The length of the subtended arc is $aa$. If this circular arc be taken as a vector, written $\mathbf{a}$, it is understood that its angle is $a$, its axis $\mathbf{A}$ and its radius $a$. A vector arc may take any position in its own circle, and has therefore one more degree of freedom than its vector axis.

Vector arcs need not be confined to arcs of circles, but whether the extension to other curves would be of any particular value remains to be seen. A rough classification gives the following:

(1) Straight vectors,

(2) Plane vectors, having single curvature.

A. Conic,

*a.* Circular, *b.* Elliptic, *c.* Parabolic, *d.* Hyperbolic,

B. Spiral, etc.

(3) Solid vectors, with double curvature.

**22. SUM OF CIRCULAR AND STRAIGHT VECTORS.**

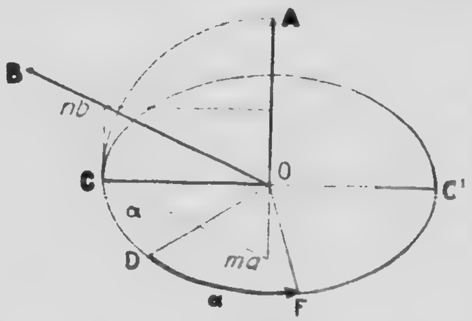



Fig. 5.

Let the plane of the arc $\mathbf{a}$ meet the plane of $\mathbf{A}$, $\mathbf{B}$, in the line $CC^1$; let $\mathbf{C}$ be so chosen that $\angle\ \mathbf{BC}$ is not greater than $90°$, i. e., so that $n$ is positive; and let $a = c$.

Let $\quad \mathbf{C} = m\mathbf{A} + n\mathbf{B}$.

Then from the figure

$$n^2 b^2 - m^2 a^2 = c^2 = a^2,$$

$$cos\ \mathbf{AB} = \frac{ma}{nb} = \frac{S_{ab}}{ab} \qquad (83)$$

$$\therefore n = \frac{a^2}{v},\ m = \frac{S}{v},$$

$$\therefore \mathbf{C} = \frac{1}{v}\,(S\,\mathbf{A} + a^2\,\mathbf{B}).$$

Let $\quad \angle\ COD = a^1$.

Then
$$\boldsymbol{\alpha} = \mathbf{F} - \mathbf{D} = e^{\boldsymbol{\alpha}^1+\boldsymbol{\alpha}}\,\mathbf{C} - e^{\boldsymbol{\alpha}^1}\,\mathbf{C} = (e^{\boldsymbol{\alpha}^1+\boldsymbol{\alpha}} - e^{\boldsymbol{\alpha}^1})\,\mathbf{C}$$
$$= \left\{cos\ (a^1 + a)\ +\ \boldsymbol{\alpha}\ sin\,(a^1 + a) - cos\ a^1 - \boldsymbol{\alpha}\ sin\ a^1\right\}\mathbf{C}$$
$$= 2\ sin\ \frac{a}{2}\ \left\{sin\ (a^1 + \frac{a}{2}) + \boldsymbol{\alpha}\ cos\ (a^1 + \frac{a}{2})\right\}\mathbf{C}.$$

To this $\mathbf{B}$ is readily added.

If $\mathbf{B}$ is parallel to $\mathbf{A}$, $\mathbf{C}$ is indeterminate and any radius of the $\boldsymbol{\alpha}$ circle may be taken as $\mathbf{C}$. In this case the sum is a point on a right helix or screw whose axis is $\mathbf{A}$. Since the addition may begin at any point of the $\boldsymbol{\alpha}$ circle, the sum is a screw vector whose radius, pitch and direction are fixed.

**23.** SUM OF TWO CIRCULAR VECTORS. Let $\boldsymbol{\alpha}, \boldsymbol{\beta}$ be two circular vectors with a common centre O; and let $\mathbf{C} = \mathbf{V}_{ab}$ be the intersection of their planes. Let $\angle\ C\,\mathbf{B}_0 = \beta^1$, $\angle\ C\,\mathbf{A}_0 = a^1$.

Then
$$\boldsymbol{\alpha} = \mathbf{A}_1 - \mathbf{A}_0 = (e^{\boldsymbol{\alpha}^1+\boldsymbol{\alpha}} - e^{\boldsymbol{\alpha}^1})\,a\,\boldsymbol{\alpha},$$
$$\boldsymbol{\beta} = \mathbf{B}_1 - \mathbf{B}_0 = (e^{\boldsymbol{\beta}^1+\boldsymbol{\beta}} - e^{\boldsymbol{\beta}})\,b\boldsymbol{\alpha}.$$

Any third circular vector whose position is determined with reference to the intersection of its plane with the plane of $\boldsymbol{\alpha}$ or $\boldsymbol{\beta}$, may be similarly expressed and the sum readily found. In expanding these expressions it is convenient to remember that

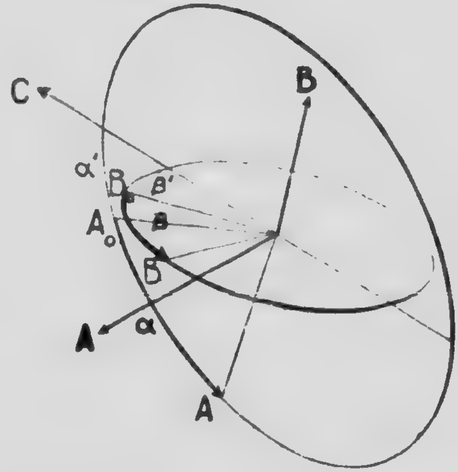



FIG. 6

when $\mathbf{C} = \mathbf{V}_{ab}$

then $\mathbf{AC} = \mathbf{V}_{ac} = S_{ab}\,\mathbf{A} - a^2\,\mathbf{B}$

and $\mathbf{BC} = \mathbf{V}_{bc} = b^2\,\mathbf{A} - S_{ab}\,\mathbf{B}$.

When $\boldsymbol{\beta} \equiv \boldsymbol{a}$ the sum is

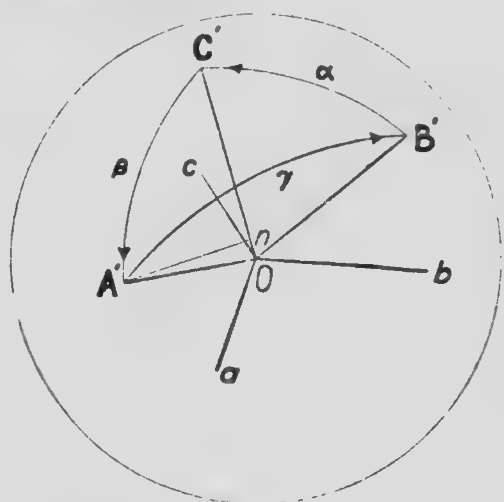
$$2\boldsymbol{a} = \boldsymbol{a}.\ 2a$$

which is a vector arc with angle $a$ and radius $2a$.

The locus of the sum of two equal vector arcs beginning at the same point of intersection, when the planes are not identical, is an ellipse.

Also $\int (\boldsymbol{a} - \boldsymbol{a})$ is a straight line.

**24.** SPHERICAL TRIANGLE. Assume a sphere of unit radius, and upon it arcs of great circles. As an illustration of vector treatment let it be required to find the relation between the sines of the angles of a spherical triangle.




Let $\boldsymbol{a}, \boldsymbol{\beta}, \boldsymbol{\gamma}$, be three circular vectors forming a spherical triangle; $\boldsymbol{a}, \boldsymbol{b}\ \boldsymbol{c}$, their vector axes; $\mathbf{A}^1$, $\mathbf{B}^1$, $\mathbf{C}^1$, the vectors from O to the angular points; A, B, C, the angles of the spherical triangle.

FIG. 7

Draw $A^1 n \perp O C^1$.

Then as vectors $O A^1 = O n + n A^1$

or $\mathbf{A}^1 = \mathbf{C}^1 + \boldsymbol{\beta} = \mathbf{C}^1 \cos\beta + \mathbf{v}_{bc^1} \sin\beta$.

Similarly $\mathbf{A}^1 = \mathbf{B}^1 - \boldsymbol{\gamma} = \mathbf{B}^1 \cos\gamma - \mathbf{v}_{cb^1} \sin\gamma$.

By inspection of the figure it is evident that in any spherical triangle

$$\boldsymbol{a} = \mathbf{v}_{b^1c^1}$$

$$\cos a = S_{b^1c^1}, \qquad \sin a = v_{b^1c^1}$$

$$\cos \mathbf{A} = -\cos(\pi - \mathbf{A}) = -S_{bc},\ \sin \mathbf{A} = v_{bc}.$$

$$\mathbf{A}^1 = \mathbf{v}_{bc} = \frac{|\,b_x\ c_y\ \mathbf{z}\,|}{v_{bc}}$$

$$= a^1{}_x\,\mathbf{x} \cdot a^1{}_y\,\mathbf{y} - a^1{}_z\,\mathbf{z}.$$

Similar equations may be written for the corresponding elements of the triangle.

From the last equation, equating coefficients of $\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}$,

$$a^1_x = \frac{|\, b_y \; c_z \,|}{v_{bc}}, \quad a^1_y = \frac{|\, b_z \; c_x \,|}{v_{bc}}, \quad a^1_z = \frac{|\, a_x \; b_y \,|}{v_{bc}}.$$

Similarly,

$$b^1_x = \frac{|\, c_y \; a_z \,|}{v_{ca}}, \quad b^1_y = \frac{|\, c_z \; a_x \,|}{v_{ca}}, \quad b^1_z = \frac{|\, c_x \; a_y \,|}{v_{ca}}.$$

Then $\quad cos\ \gamma = S_{a^1b^1} = \dfrac{S_{bc}\, S_{ca} - S_{ab}}{v_{bc}\; v_{ac}}$

and $\quad \dfrac{sin\ \gamma}{sin\ C} = \dfrac{\sqrt{1 - S^2_{a^1b^1}}}{v_{ab}} = \dfrac{\sqrt{(1 - S^2_{ab} - S^2_{bc} - S^2_{ca} + 2\, S_{ab}\, S_{bc}\, S_{ca})}}{v_{ab}\; v_{bc}\; v_{ca}}.$

The last expression is symmetrical in $a, b, c$, and therefore

$$\frac{sin\ a}{sin\ A} = \frac{sin\ \beta}{sin\ B} = \frac{sin\ \gamma}{sin\ C}.$$

**25.** CONIC VECTORS are expressible in terms of the radius vector from the focus to each extremity of the segment of the curve.

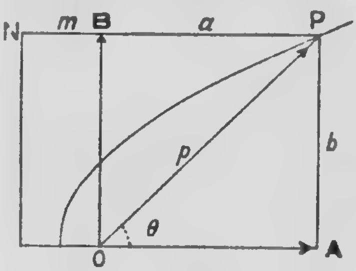



FIG. 8

Let $\mathbf{A}$ be the axis of a conic, O its focus, N its directrix, $\mathbf{P}$ the radius vector, $a, b$ the coordinat s of $\mathbf{P}$ with reference to $\mathbf{A}$ and $\mathbf{B}$; and let $p = e\ (a + m)$, where $e = \dfrac{1}{c}$ is the eccentricity.

Then $\qquad a = cp - m$

$$b^2 = p^2 - a^2 = p^2\,(1 - c^2) + 2cmp - m^2.$$

$$\therefore \mathbf{P} = \mathbf{A} + \mathbf{B}$$

$$= (cp - m)\,\boldsymbol{a} + \boldsymbol{b}\sqrt{[p^2\,(1 - c^2) + 2cmp - m^2]}.$$

The conic vector from $P_o$ to P is $\mathbf{P} - \mathbf{P}_o$.

P may also be expressed in terms of $a, b$, or $\theta$.

Thus $\qquad a = p\ cos\ \theta, \qquad cp = a + m$

$$b = p\ sin\ \theta, \qquad = p\ cos\ \theta + m,$$

$$p = \frac{m}{c - cos\ \theta}.$$

$$\therefore \mathbf{P} = \frac{m}{c - cos\ \theta}\ (\boldsymbol{a}\ cos\ \theta + \boldsymbol{b}\ sin\ \theta).$$

If $p$ is a constant, $c = m = \infty$, then for the circle

$$\mathbf{P} = p\ (\boldsymbol{a} \cos \theta + \boldsymbol{b} \sin \theta).$$

Similarly in any conic

$$\mathbf{P} = \mathbf{A} + \frac{\boldsymbol{b}}{c} \sqrt{\left\{ (a + m)^2 - a^2 c^2 \right\}}.$$

Also $$\mathbf{P} = \frac{m \pm c\sqrt{[m^2 + b^2 (c^2 - 1)]}}{c^2 - 1}\ \boldsymbol{a} + \mathbf{B},$$

but when $c = 1$, in the parabola,

$$\mathbf{P} = \frac{b^2 - m^2}{2m}\ \boldsymbol{a} + \mathbf{B}.$$

We have then an expression for any conic vector as the difference of two straight vectors, $\mathbf{P} = \mathbf{P}_0$; which may be expressed in terms of either of the variables, $a$, $b$ or $\theta$.

The sum of two or more conic vectors would express approximately for a short distance the course of a body moving under gravitational forces from two or more sources. Whether this method of calculating would be an improvement on present methods I am not prepared to say.

Multiplication of a straight vector by a vector arc involves double curvature, and the locus of such a product is a convenient form by which to express solid vectors (§21). Again the utility is problematical.

**26.** DIFFERENTIATION OF STRAIGHT VECTORS. Any vector, $\mathbf{A}$, may vary in length and in direction. Its variation may be expressed in terms of $a$ for length and $\boldsymbol{a}$ for direction; or it may be expressed in terms of the components $\mathbf{A}_x$, $\mathbf{A}_y$, $\mathbf{A}_z$.

Since the infinitesimal increments of a vector are also vectors, it is evident that by vector addition

$$\begin{aligned} d\mathbf{A} &= d_x \mathbf{A} + d_y \mathbf{A} + d_z \mathbf{A} \\ &= \boldsymbol{x}\, da_x + \boldsymbol{y}\, da_y + \boldsymbol{z}\, da_z \qquad (1) \end{aligned}$$

since $\boldsymbol{x}$, $\boldsymbol{y}$, $\boldsymbol{z}$, are absolute constants.

Also, $$\begin{aligned} d\mathbf{A} &= d_{\boldsymbol{a}} \mathbf{A} + d_a \mathbf{A} \\ &= a\, d\boldsymbol{a} + \boldsymbol{a}\, da. \qquad (2) \end{aligned}$$

It follows from (1) that the differential of a vector is the sum of the differentials of its components, and hence that differentiation is distributive over vector terms.

It follows from (2) that the ordinary rule for differentiation of a product holds true for any unit vector and its tensor, and hence for any product of a tensor and a vector.

**27.** To find the differential coefficient of a vector product, $\mathbf{A}^n\mathbf{B}$.

$$\mathbf{A}^n\mathbf{B} = a^{n-1}\,(S\boldsymbol{\alpha}\ vers\ \theta + a\,\mathbf{B}\cos\theta + \mathbf{V}\sin\theta).$$

Differentiating both sides of the equation with respect to $\theta$, $= n\frac{\pi}{2}$,

$$\frac{d}{d\theta}\mathbf{A}^n\mathbf{B} = \frac{2}{\pi}\,\mathbf{A}^n\mathbf{B}.\log a + a^{n-1}(S\boldsymbol{\alpha}\sin\theta - a\,\mathbf{B}\sin\theta + \mathbf{V}\cos\theta)$$

$$= \frac{2}{\pi}\,\mathbf{A}^n\mathbf{B}\log a + a^{n-1}(\cos\theta + \boldsymbol{\alpha}\sin\theta)\,\mathbf{V},$$

since, by multiplication, $S\boldsymbol{\alpha} - a\,\mathbf{B} = \boldsymbol{\alpha}\,\mathbf{V}$.

The last term of the differential coefficient may also be written $a^{n-1}e^{\boldsymbol{\alpha}\theta}\,\mathbf{V}$. It is a tensor and rotor product of $\mathbf{V}$ (the vector normal of $\mathbf{A}$ and $\mathbf{B}$), whose rotation is about $\mathbf{A}$. $e^{\boldsymbol{\alpha}\theta}\,\mathbf{V}$ expresses the rotation of $\mathbf{V}$ in the plane perpendicular to $\mathbf{A}$. $a^{n-1}e^{\boldsymbol{\alpha}\theta}\,\mathbf{V}$ traces a spiral in this plane, and as a vector it gives at any point the direction and rate of motion in this plane made by the point $\mathbf{B}$, supplementary to the increase in the length of $\mathbf{B}$.

The term $\frac{2}{\pi}\,\mathbf{A}^n\mathbf{B}\log a$ is a multiple of the vector product, and for any given value of $a$ it expresses the rate and direction of the increase, in length only, of that product.

The sum of the two terms gives the rate and direction of the motion of the point $\mathbf{B}$ for unit increase in $\theta$. It is the vector tangent to the curve traced by $\mathbf{A}^n\mathbf{B}$, namely, the curve $/\mathbf{A}^n\mathbf{B}$.

EXAMPLE.—Find the tangent where the flat spiral $(a\,\mathbf{x})^n.\,b\,\mathbf{y}$ cuts the Y axis. The tangent is

$$\mathbf{T} = \frac{d}{d\theta}(a\,\mathbf{x})^n\,b\,\mathbf{y},$$

$$= a^n b\left[\left(\frac{2}{\pi}\log a.\cos\theta - \sin\theta\right)\mathbf{y} + \left(\frac{2}{\pi}\log a.\sin\theta - \cos\theta\right)\mathbf{z}\right]$$

At the starting point $\theta = o$, and

$$\mathbf{T}_0 = b\left(\frac{2}{\pi}\log a.\,\mathbf{y} + \mathbf{z}\right).$$

When $n = 2$, $\theta = \pi$, and

$$\mathbf{T}_2 = -\,a^2\,\mathbf{T}_0.$$

When $n = 4$, $\theta = 2\pi$, and

$$\mathbf{T}_4 = a^4\,\mathbf{T}_0 = -\,a^2\,\mathbf{T}_2 .$$

This vector tangent makes at all times a constant angle with its radius, and its length gives the velocity of the generating point when the angular velocity is unity.

**28.** CURVATURE. If $l$ be the length of a curve, and $\mathbf{T}$ the vector tangent, the curvature $\mathbf{K}$ is $\frac{d\mathbf{T}}{dl}$, and the radius of curvature is

$$\mathbf{R} = \frac{-\mathbf{K}}{k^2} .$$

**29.** LINEAR LOCI are loci having only one degree of freedom; lines or discrete points. A few examples are given:

$_{a_0}l^{a}\,\mathbf{A} + \mathbf{B}$ is any part of the straight line drawn from the point $\mathbf{B}$ in the direction $\mathbf{A}$.

$_{n=0}l^{n=4}\,\mathbf{A}^n\mathbf{B} + \mathbf{C}$, when $\mathbf{A} \perp \mathbf{B}$, is a circle with centre $\mathbf{C}$, radius $b$, and plane perpendicular to $\boldsymbol{a}$.

$_0l^{2\pi}(\mathbf{A}\,cos\,\theta + \mathbf{B}\,sin\,\theta) + \mathbf{C}$ is an ellipse parallel to the $\mathbf{A}\,\mathbf{B}$ plane.

$_0l^{n}(\mathbf{A}^n\mathbf{B} + n\mathbf{C})$ includes a variety of curves.
If $\mathbf{A} \perp \mathbf{B}$, $\mathbf{C} \parallel \mathbf{A}$, and $a = 1$, the locus is a helix.
If $\mathbf{C} \perp \mathbf{A}$ the locus varies from a circle (when $c = o$) to a straight line (when $c = \infty$), passing thru the cycloid. In other positions of $\mathbf{C}$ the helix is acute angled. When $a > 1$ the curves are expanding and when $a < 1$ diminishing.

**30.** EXAMPLES OF SURFACE LOCI.

$_0l^{a}\,_0l^{b}(\mathbf{A} + \mathbf{B}) + \mathbf{C}$ is a parallelogram whose adjacent sides $\mathbf{A}, \mathbf{B}$, start at the point $\mathbf{C}$. Its diagonals are $\mathbf{A} \pm \mathbf{B}$. Its area is $ab\;sin\;\mathbf{A}\,\mathbf{B} = v$. (§10).

${}_{0}l^{2}\,{}_{0}l^{4}\,\boldsymbol{b}^{n}\,\boldsymbol{a}^{m}\,\mathsf{B}$ is a closed surface, spherical if $\boldsymbol{a} \perp \boldsymbol{b}$, with radius $b$.

${}_{0}l^{4}\,{}_{0}l^{b}\,\boldsymbol{a}^{n}\,\mathsf{B}$ is the conical surface traced by $\mathsf{B}$ as it is turned about $\mathsf{A}$.

${}_{0}l^{a}\,{}_{0}l^{b}(\mathsf{A} + (1-a)\,\mathsf{B})$ is the triangle OAB.

**31.** EXAMPLES OF SOLID LOCI.

$lll\,(\mathsf{A} + \mathsf{B} + \mathsf{C}) + \mathsf{D}$ is any parallelopiped.

Its diagonals are $\mathsf{A} + \mathsf{B} + \mathsf{C}$, $\mathsf{A} + \mathsf{B} - \mathsf{C}$, $\mathsf{A} - \mathsf{B} + \mathsf{C}$, $-\mathsf{A} + \mathsf{B} + \mathsf{C}$. Its volume is $|\,a_x\, b_y\, c_z\,|$. If $\mathsf{P} = \mathsf{V}_{ab}$ and $\mathsf{Q} = \mathsf{V}_{ac}$, the dihedral angle, $a$, over the edge $\mathsf{A}$ is found from the equation $S_{pq} = p\,q \cos a$.

${}_{0}l^{2}\,{}_{0}l^{4}\,{}_{a_0}l^{a}\,\boldsymbol{a}^{n}\,\boldsymbol{b}^{m}\,\mathsf{A}$ is a shell, spherical if $\mathsf{A} \perp \mathsf{B}$.

${}_{0}l^{4}\,\boldsymbol{a}^{n}\left\{{}_{0}l^{4}\,\boldsymbol{b}^{m}\;{}_{c_0}l^{c}\,\mathsf{C} + \mathsf{R}\right\}$ is a hollow annulus

if $\mathsf{B} \perp \mathsf{C}$, $\mathsf{R} \perp \mathsf{C}$, $\mathsf{A} \perp \mathsf{R}$, $\mathsf{A} \perp \mathsf{B}$.

**32.** THE REGION COMMON to two loci is found by equating the coefficients of $\boldsymbol{x}$, $\boldsymbol{y}$, $\boldsymbol{z}$, in the expressions for the loci. If these equations are consistent, giving real values for the variables, the limits thus found are inserted in either of the loci to give the required locus of intersection.

EXAMPLE 1.—Find the region common to the straight line

${}_{0}l^{n}\,n\boldsymbol{x} + \tfrac{1}{2}\boldsymbol{y}$, and the curve

${}_{0}l^{a}\left\{a\,\boldsymbol{x} + (2\boldsymbol{x} + \boldsymbol{y}) \sin a\right\}$

Equating coefficients,

$n = a + 2 \sin a$

$\tfrac{1}{2} = \sin a.$

Whence $n = a + 1 = \arcsin \tfrac{1}{2} + 1.$

Inserting these values, both loci become

${}_{0}l^{a}(\tfrac{1}{2}\,\boldsymbol{y} + \boldsymbol{x} \arcsin \tfrac{1}{2})$

which is a row of discrete points parallel to $x$.

EXAMPLE 2.—Find what part of the helix

$$l_0^n (x^n y + 3n x) \equiv l_0^n \left(3nx + y \cos\frac{n\pi}{2} + z \sin\frac{n\pi}{2}\right)$$

is within the figure

$$l_0^4 l_0^c l_0^b (y^m c z + bx - y) \equiv l_0^4 l_0^c l_0^b \left\{ x \left(c \sin\frac{m\pi}{2} + b\right) - y + z\, c \cos\frac{m\pi}{2} \right\}.$$

Equating coefficients of $x$, $y$, $z$,

(1) $3n = c \sin\dfrac{m\pi}{2} + b$

(2) $\cos\dfrac{n\pi}{2} = -1$, $\therefore \sin\dfrac{n\pi}{2} = 0$, and $n = 2, 6, 10 \ldots\ldots$

(3) $c \cos\dfrac{m\pi}{2} = \sin\dfrac{n\pi}{2} = 0$,

If $c = 0$, $3n = b$.

If $\cos\dfrac{m\pi}{2} = 0$, $\sin\dfrac{m\pi}{2} = \pm 1$, and since $c$ is positive $3n = b + c$.

Inserting these values in the locus of the helix we get for the intersection a row of points

$l\, 3nx - y$, where $n$ has the values

2, 6, 10..........up to $\dfrac{b+c}{3}$ .

EXAMPLE 3.—Find the intersection of the plane

$$ll\,(mx + nz) + 3x$$

with the solid

$$l_0^2 l_0^a l_0^b x^n (ax + by) \equiv l_0^{\pi} l_0^a l_0^b \{ ax + by \cos\theta + bz \sin\theta \}.$$

Equating coefficients of $x$, $y$, $z$,

(1) $m + 3 = a$, or $m = a - 3$.

(2) $b \cos\theta = 0$, $\therefore b = 0$, or $\sin\theta = \pm 1$.

(3) $n = b \sin\theta$, $= 0$ or $\pm b$.

$\therefore$ $n = \pm b$.

Substituting in the locus of the plane we get for the intersection the parallelogram

$$l_0^a l_0^b (ax \pm bz).$$

EXAMPLE 4.—Find the locus of the intersection of the cube

$$\int_0^1\int_0^1\int_0^1 (a_x\mathbf{x} + a_y\mathbf{y} + a_z\mathbf{z})$$

with a plane which cuts its diagonal

$\mathbf{A} = \mathbf{x} + \mathbf{y} + \mathbf{z}$ perpendicularly.

Let $\mathbf{B} = \mathbf{x} - \mathbf{y}$

be one vector in the perpendicular plane, and

$$\mathbf{C} = \mathbf{V}_{ab} = \mathbf{x} + \mathbf{y} - 2\mathbf{z}$$

the other. The plane is

$$\int\int (l\,\mathbf{B} + m\mathbf{C} + n\mathbf{A})$$

where $n$ is an arbitrary constant expressing the fractional distance from O to the point where the diagonal is cut.

Equate coefficients of $\mathbf{x}$, $\mathbf{y}$, $\mathbf{z}$, in the two loci,

$$a_x = l + m + n$$
$$a_y = -l + m + n$$
$$a_z = -2m + n.$$

Therefore $\quad l = \dfrac{a_x - a_y}{2}$

$$m = \frac{a_x + a_y - 2a_z}{6}$$

$$3n = a_x + a_y + a_z.$$

If $n = 0$ the plane goes thru O. Since $a_x, a_y, a_z$. are all positive and the sum zero, each of them is zero, and the locus of intersection is the point O.

If $n = 1$ the point of intersection is $\mathbf{A}$.

If $n = \frac{1}{3}$, so that $a_x + a_y + a_z = 1$, while each varies between 0 and 1 subject to this condition, the locus is an equilateral triangle whose corners are found by giving to $a_x, a_y, a_z$, separately the maximum value, 1, in the expanded expression for the plane

$$\int_0^1\int_0^1\int_0^1\left\{\frac{a_x - a_y}{2}\mathbf{B} + \frac{a_x + a_y - 2a_z}{6}\mathbf{C}\right\} + \tfrac{1}{3}\mathbf{A}.$$

If $n = \frac{2}{3}$ the locus is a similar triangle.

If $n = \frac{1}{2}$ the locus a regular hexagon.

**33.** PROJECTIONS. To express any vector **K** in terms of three non-coplanar vectors **A**, **B**, **C**, write

$$l\mathbf{A} + m\mathbf{B} + n\mathbf{C} = \mathbf{K}$$

$$\therefore \quad l\,a_x + m\,b_x + n c_x = k_x$$

$$l\,a_y + m\,b_y + n\,c_y = k_y$$

$$l\,a_z + m\,b_z + n\,c_z = k_z$$

$$\therefore \quad l = \frac{|\,k_x\,b_y\,c_z\,|}{|\,a_x\,b_y\,c_z\,|}, \quad m = \frac{|\,k_x\,c_y\,a_z\,|}{|\,a_x\,b_y\,c_z\,|}, \quad n = \frac{|\,k_x\,a_y\,b_z\,|}{|\,a_x\,b_y\,c_z\,|}.$$

If we now write $n = 0$, $l\mathbf{A} + m\,\mathbf{B}$ is the projection of **K**, made parallel to **C**, upon the plane of **A**, **B**.

If **A** and **B** only are given, and the projection is desired of **K** perpendicularly upon **A**, **B**, take $\mathbf{C} = \mathbf{V}_{ab} = |\,a_x\,b_y\,\boldsymbol{z}\,|$, and proceed as before.

To project **K** in the direction of **C** upon a plane perpendicular to **C**, take any vector **A**, $\perp$ **C**, so that $S_{ac} = 0$,

as, $\quad \mathbf{A} = c_y\boldsymbol{x} - c_x\,\boldsymbol{y}$

and a second vector **B**, $= \mathbf{V}_{ac}$,

$$\mathbf{B} = \begin{vmatrix} c_x & c_y & c_z \\ c_y & -c_x & 0 \\ \boldsymbol{x} & \boldsymbol{y} & \boldsymbol{z} \end{vmatrix}.$$

Then express **K** in terms of **A**, **B**, **C**, as before.

The most general form of a locus is $\boldsymbol{lll}\,\mathbf{K} + \mathbf{M}$, which is projected in the same way.

EXAMPLE.—Project upon the YZ plane and parallel to **D** the helix

$$\boldsymbol{l}\,\mathbf{B} = {}_0\boldsymbol{l}^n (\boldsymbol{x}^n b\,\boldsymbol{y} + a\,n\,\boldsymbol{x})$$

$$= {}_0\boldsymbol{l}^n \left\{ b\,(\boldsymbol{y} \cos\theta + \boldsymbol{z} \sin\theta) + a\,n\,\boldsymbol{x} \right\}.$$

Let $\mathbf{B} = l\,\boldsymbol{y} + m\boldsymbol{z} + r\mathbf{D}$.

Equating the coefficients of $\boldsymbol{x}$, $\boldsymbol{y}$, $\boldsymbol{z}$,

$$r\,d_x = a\,n$$

$$l + r d_y = b \cos\theta$$

$$m + r d_z = b \sin\theta.$$

$$\therefore \quad \mathbf{B}' = \boldsymbol{y}\left\{ b\cos\theta - a\,n\,\frac{d_y}{d_x} \right\} + \boldsymbol{z}\left\{ b\sin\theta - a\,n\,\frac{d_z}{d_x} \right\}$$

the locus of which is the required projection.

**34. PLANE ALGEBRA.** Every vector in the X Y plane is of the form

$$\mathbf{A} = a_x \mathbf{x} + a_y \mathbf{y}$$
$$= a_x \mathbf{x} + a_y \mathbf{z}\mathbf{x}$$
$$= (a_x + \mathbf{z}\, a_y)\mathbf{x}.$$

Since $\mathbf{x}$ is a part of every vector expression of this form, it may be omitted. The remaining form, $a_x + \mathbf{z}\, a_y$, is a complex number. Since $\mathbf{z}^2$ as a rotor is equivalent to -1, we may write this tensor-rotor in the common form $a + ib$ (where $i^2 = -1$), whose properties are well known.

Again, any vector in the X Y plane may be expressed as a $\mathbf{z}$-product, thus,

$$\mathbf{A} = a\, \mathbf{z}^n\, \mathbf{x}$$
$$= a\,(\cos\theta + \mathbf{z}\sin\theta)\, \mathbf{x}$$
$$= a\, \epsilon^{\mathbf{z}\theta}\, \mathbf{x}.$$

Omitting $\mathbf{x}$ as before we have left the other two forms of the complex number.

Vector multiplication in the X Y plane with any other rotor than $\mathbf{z}$ gives in general imaginary products, i.e., products lying outside of that plane.

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## FOUR-SPACE ALGEBRA

**35.** In four-space there are, by definition, four mutually perpendicular axes, X, Y, Z, U. These are so selected that they multiply in circular order, as in 3-space. Each vector is now fully defined by four components. Vectors are added and subtracted as in 3-space.

As in §3 it may be shown that

$$S_{ab} \equiv a_x b_x + a_y b_y + a_z b_z + a_u b_u = ab \cos \mathbf{AB},$$

where $S_{ab}$ is, as before, the sum of the $a\,b$ products.

Evidently also when $S_{ab} = 0$, $\mathbf{A} \perp \mathbf{B}$.

**36. MULTIPLICATION** in 4-space is defined as rotation about the plane* of the multiplying vectors, thru a right angle in the positive direction. The planes of rotation are wholly† perpendicular to the axial plane.

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*Rotation is essentially plane motion. In a 2-flat the axis of rotation is a point. In a 3-flat the axis is a line. In a 4-flat the axis is a plane.

†It is evident from §35, that in 4-space absolutely perpendicular planes exist. For $\mathbf{A} = a_x \mathbf{x} + a_y \mathbf{y}$ is any vector in the X Y plane, and $\mathbf{B} = b_z \mathbf{z} + b_u \mathbf{u}$ is any vector in the Z U plane. Since $S_{ab} = 0$, $\mathbf{A} \perp \mathbf{B}$. That is to say, every vector in the X Y plane is perpendicular to every vector in the Z U plane.

Multiplication of **C** by **AB**, is written $\overline{\mathbf{AB}}\mathbf{C}$, and is defined as

(1) Rotation of **C** thru 90° in the positive direction about the plane **A B**, and

(2) Simultaneous extension to the length $abc$.

By definition, $\overline{\boldsymbol{x}\boldsymbol{y}}\boldsymbol{z} = \boldsymbol{u}$, $\overline{\boldsymbol{y}\boldsymbol{z}}\boldsymbol{u} = \boldsymbol{x}$, $\overline{\boldsymbol{z}\boldsymbol{u}}\boldsymbol{x} = \boldsymbol{y}$, $\overline{\boldsymbol{u}\boldsymbol{x}}\boldsymbol{y} = \boldsymbol{z}$.

Remembering that the plane of rotation is perpendicular to the axial plane it becomes evident that

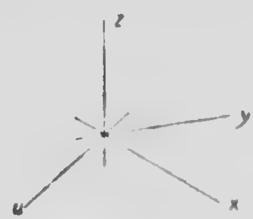


FIG. 9

$$\overline{\boldsymbol{x}\boldsymbol{y}}\boldsymbol{u} = \bar{\boldsymbol{z}},\quad \overline{\boldsymbol{y}\boldsymbol{z}}\boldsymbol{x} = \bar{\boldsymbol{u}},\quad \overline{\boldsymbol{z}\boldsymbol{u}}\boldsymbol{y} = \bar{\boldsymbol{x}},\quad \overline{\boldsymbol{u}\boldsymbol{x}}\boldsymbol{z} = \bar{\boldsymbol{y}}$$

$$\overline{\boldsymbol{x}\boldsymbol{y}}\bar{\boldsymbol{z}} = \bar{\boldsymbol{u}},\quad \overline{\boldsymbol{y}\boldsymbol{z}}\boldsymbol{u} = \bar{\boldsymbol{x}},\quad \overline{\boldsymbol{z}\boldsymbol{u}}\bar{\boldsymbol{x}} = \bar{\boldsymbol{y}},\quad \overline{\boldsymbol{u}\boldsymbol{x}}\bar{\boldsymbol{y}} = \boldsymbol{z}$$

$$\overline{\boldsymbol{x}\boldsymbol{y}}\bar{\boldsymbol{u}} = \boldsymbol{z},\quad \overline{\boldsymbol{y}\boldsymbol{z}}\bar{\boldsymbol{x}} = \boldsymbol{u},\quad \overline{\boldsymbol{z}\boldsymbol{u}}\bar{\boldsymbol{y}} = \boldsymbol{x},\quad \overline{\boldsymbol{u}\boldsymbol{x}}\bar{\boldsymbol{z}} = \boldsymbol{y}$$

Coplanar vectors are unchanged in position by 4-space multiplication, because the whole axial plane is unmoved,

$$\overline{\boldsymbol{x}\boldsymbol{y}}\boldsymbol{x} = \boldsymbol{x}$$

$$\overline{\boldsymbol{x}\boldsymbol{y}}\,(a_x\boldsymbol{x} + a_y\boldsymbol{y}) = a_x\boldsymbol{x} + a_y\boldsymbol{y}.$$

## 37. MULTIPLICATION BY PERPENDICULAR VECTORS.

Let $\mathbf{A} \perp \mathbf{B} \perp \mathbf{C}$, and let $\overline{\mathbf{AB}}\mathbf{C} = \mathbf{W}$.

Then $S_{aw} = S_{bw} = S_{cw} = 0$

and $w^2 = w_x^2 + w_y^2 + w_z^2 + w_u^2 = a^2 b^2 c^2$.

Solving for $w_x$, $w_y$, $w_z$, $w_u$, and collecting,

$$\mathbf{W}_{abc} \equiv \begin{vmatrix} a_x & a_y & a_z & a_u \\ b_x & b_y & b_z & b_u \\ c_x & c_y & c_z & c_u \\ \boldsymbol{x} & \boldsymbol{y} & \boldsymbol{z} & \boldsymbol{u} \end{vmatrix} \equiv |\, a_x\, b_y\, c_z\, \boldsymbol{u} \,|\,.$$

Also

$$w^2 \equiv |\, a_y\, b_z\, c_u \,|^2 + |\, a_x\, b_z\, c_u \,|^2 + a_x\, b_y\, c_u \,|^2 + |\, a_x\, b_y\, c_z \,|^2$$

$$\equiv \begin{vmatrix} S_{aa} & S_{ab} & S_{ac} \\ S_{ba} & S_{bb} & S_{bc} \\ S_{ca} & S_{cb} & S_{cc} \end{vmatrix} = a^2\, b^2\, c^2.$$

**38.** $\mathbf{C}$ is coplanar with $\mathbf{A}$, $\mathbf{B}$, when

$$\mathbf{C} = m\mathbf{A} + n\mathbf{B}.$$

Writing the four equations of coordinates, and eliminating $m$ and $n$, we get the coplanar equations

$$| a_x\, b_y\, c_z | = 0$$
$$| a_y\, b_z\, c_x | = 0.$$

**39.** To find the perpendicular $\mathbf{N}_2$ from the point $\mathbf{B}$ to the vector $\mathbf{A}$.

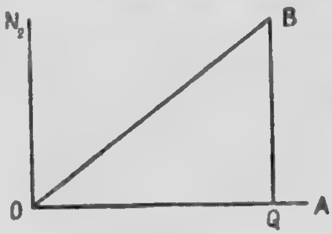


Fig. 10

Let Q B be the positive direction of $\mathbf{N}_2$.

Then $q = \mathrm{OQ} = b \cos \mathbf{AB} = \dfrac{S_{ab}}{a}$.

$$\therefore\ \mathbf{Q} = \frac{S_{ab}}{a^2}\,\mathbf{A}.$$

$$\therefore\ \mathbf{N}_2 = \mathbf{B} - \mathbf{Q} = \begin{vmatrix} S_{aa} & S_{ab} \\ \mathbf{A} & \mathbf{B} \end{vmatrix} \div a^2.$$

$$n^2 = b^2 - q^2 = \begin{vmatrix} S_{aa} & S_{ab} \\ S_{ba} & S_{bb} \end{vmatrix} \div a^2 = \frac{v^2}{a^2}.$$

These forms of $\mathbf{N}_2$, $n^2$, $\mathbf{Q}$, $q$, are identical in space of four, three and two dimensions, and evidently for space of all dimensions.

In a 2-flat

$$n = \begin{vmatrix} a_x & a_y \\ b_x & b_y \end{vmatrix} \div a.$$

**40.** To find the perpendicular $\mathbf{N}_3$ from $\mathbf{C}$ to the $\mathbf{AB}$ plane.

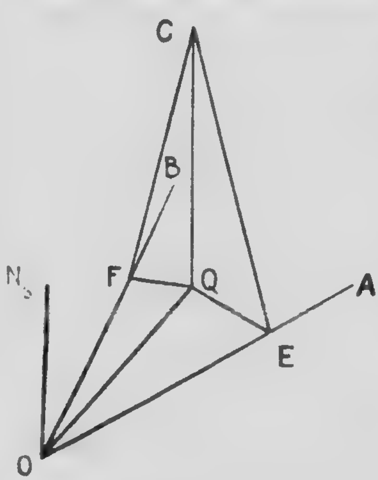


Fig. 11

Let Q C be the positive direction of $\mathbf{N}_3$.
Draw Q E $\perp$ $\mathbf{A}$, Q F $\perp$ $\mathbf{B}$, join C F, C E.

Then $\quad \mathrm{OE} = \dfrac{S_{aq}}{a} = \dfrac{S_{ac}}{a}$

$\therefore\ S_{aq} = S_{ac}$ ..............................(1)

and $\quad \mathrm{OF} = \dfrac{S_{bq}}{b} = \dfrac{S_{bc}}{b}$.

$\therefore\ S_{bq} = S_{bc}$ ..............................(2)

Since $\mathbf{A}$, $\mathbf{B}$, $\mathbf{Q}_3$, are coplanar

$| a_x\, b_y\, q_z | = 0$ ..............................(3)

$| a_y\, b_z\, q_x | = 0$ ..............................(4).

Solving for the coordinates of $\mathbf{Q}_3$, and collecting terms,

$$\mathbf{Q}_3 = (b^2\, S_{ac} - S_{ab}\, S_{bc})\, \frac{\mathbf{A}}{v^2} + (a^2\, S_{bc} - S_{ab}\, S_{ac})\, \frac{\mathbf{B}}{v^2},$$

$$q_3^2 = q_x^2 + q_y^2 + q_z^2 + q_u^2$$

$$= (a^2\, S_{bc}^2 + b^2\, S_{ac}^2 - 2\, S_{ab}\, S_{ac}\, S_{bc})\, \frac{1}{v^2},$$

where $\qquad v^2 = a^2 b^2 - S_{ab}^2$.

Then

$$\mathbf{N}_3 = \mathbf{C} - \mathbf{Q}_3 = \begin{vmatrix} S_{aa} & S_{ab} & S_{ac} \\ S_{ba} & S_{bb} & S_{bc} \\ \mathbf{A} & \mathbf{B} & \mathbf{C} \end{vmatrix} \div \begin{vmatrix} S_{aa} & S_{ab} \\ S_{ba} & S_{bb} \end{vmatrix}$$

$$n_3^2 = c^2 - q_3^2 = \begin{vmatrix} S_{aa} & S_{ab} & S_{ac} \\ S_{ba} & S_{bb} & S_{bc} \\ S_{ca} & S_{cb} & S_{cc} \end{vmatrix} \div \begin{vmatrix} S_{aa} & S_{ab} \\ S_{ba} & S_{bb} \end{vmatrix}$$

$$= \frac{w^2}{v^2}.$$

These forms are identical for 3-space, and apparently for all spac above it.

In 3-space also $\qquad n_3 = \dfrac{|\, a_x\, b_y\, c_z \,|}{v}$.

**41.** To find the normal $\mathbf{N}_4$, from $\mathbf{D}$ to the 3-flat of $\mathbf{A}, \mathbf{B}, \mathbf{C}$.

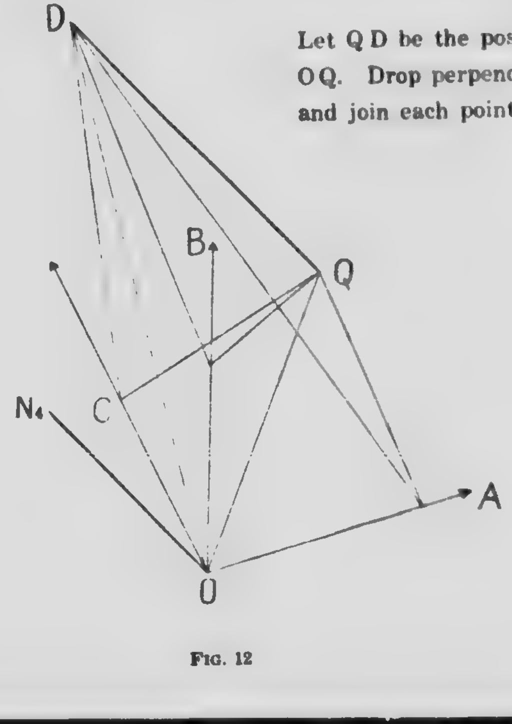


FIG. 12

Let Q D be the positive directicn of $\mathbf{N}_4$. Joi O Q. Drop perpendiculars from $\mathbf{Q}_4$ on $\mathbf{A}, \mathbf{B}, \mathbf{C}$ and join each point of intersection with $\mathbf{D}$.

Then it is evident as i §40 that

$S_{aq} = S_{ad}$ ............ (1

$S_{bq} = S_{bd}$ ........ (2

$S_{cq} = S_{cd}$ ......... (3

Since $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{Q}_4$, a all in the same 3-flat ar therefore all perpe dicular to $\mathbf{N}_4$

$S_{an} = S_{bn} = S_{cn} = S_{qn} =$

Eliminating the $n$'s w get the cosolid equati

$|\, a_x\, b_y\, c_z\, q_u \,| = 0$ ... (4

Solving for $q_a$ etc. and collecting terms

$$\mathbf{Q}_4 = \{ S_{ad} [\mathbf{A}(S_{bc}^2 - b^2c^2) + \mathbf{B}(c^2 S_{ab} - S_{ca} S_{bc}) + \mathbf{C}(b^2 S_{ac} - S_{ab} S_{bc})]$$
$$+ S_{bd} [\mathbf{B}(S_{ac}^2 - a^2c^2) + \mathbf{C}(a^2 S_{bc} - S_{ab} S_{ac}) + \mathbf{A}(c^2 S_{ab} - S_{bc} S_{ac})]$$
$$+ S_{cd} [\mathbf{C}(S_{ab}^2 - a^2b^2) + \mathbf{A}(b^2 S_{ac} - S_{bc} S_{ab}) + \mathbf{B}(a^2 S_{bc} - S_{ac} S_{ab})] \} \div w^2$$

Therefore

$$\mathbf{N}_4 = \mathbf{D} - \mathbf{Q}_4 = \begin{vmatrix} S_{aa} & S_{ab} & S_{ac} & S_{ad} \\ S_{ba} & S_{bb} & S_{bc} & S_{bd} \\ S_{ca} & S_{cb} & S_{cc} & S_{cd} \\ \mathbf{A} & \mathbf{B} & \mathbf{C} & \mathbf{D} \end{vmatrix} \div \begin{vmatrix} S_{aa} & S_{ab} & S_{ac} \\ S_{ba} & S_{bb} & S_{bc} \\ S_{ca} & S_{cb} & S_{cc} \end{vmatrix}.$$

$$n_4^2 = \frac{|\, S_{aa}\ S_{bb}\ S_{cc}\ S_{dd} \,|}{|\, S_{aa}\ S_{bb}\ S_{cc} \,|} = \frac{|\, a_x\ b_y\ c_z\ d_u \,|^2}{w^2}.$$

**42. RELATION OF $\mathbf{N}_4$ TO THE RECTOR W.** Since $\mathbf{N}_4$ and **W** are each perpendicular to the 3-flat of **A**, **B**, **C**, they differ only in their tensors,

Hence $$\mathbf{N}_4 = \frac{n_4}{w}\mathbf{W} = \frac{|\, a_x\ b_y\ c_z\ d_u \,|}{w^2}\mathbf{W}.$$

Similarly, in 3-space

$$\mathbf{N}_3 = \frac{n_3}{v}\mathbf{V} = \frac{|\, a_x\ b_y\ c_z \,|}{v^2}\mathbf{V}.$$

And in 2-space

$$\mathbf{N}_2 = \frac{n_2}{f}\mathbf{F} = \frac{|\, a_x\ b_y \,|}{f^2}\mathbf{F},$$

when $\mathbf{F} \equiv \begin{vmatrix} a_x & a_y \\ x & y \end{vmatrix}$ is the perpendicular to **A**.

The forms for $\mathbf{N}_3$, $\mathbf{N}_2$, $n_3^2$, $n_2^2$, may be obtained by suppressing rows and columns in the determinant forms of $\mathbf{N}_4$, $n_4^2$. It is evident that we have here a correspondence between the geometric space-form for a perpendicular and the algebraic space-form or matrix, which is true for all space.

**43.** To find the product $\overline{\mathbf{AB}}^n\mathbf{C}$ when $n$ is real.

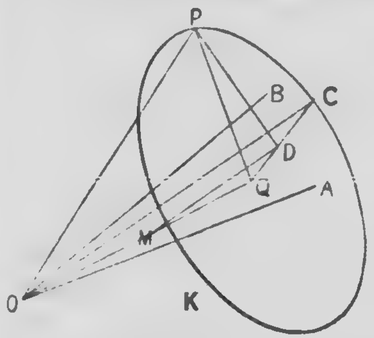


Fig. 13

Let CPK be the circle of rotation of the point **C**, and let Q be its centre in the AB plane.

Join QO, QC, QP.

Let OP be the position of OC after rotation, so that $OP = \overline{\mathbf{ab}}^n\mathbf{C}$.

Draw PD $\perp$ QC, DM $\parallel$ CO.

The angle $CQP = \theta = n\frac{\pi}{2}$.

Then since PD, being in the plane of rotation, is perpendicular to the **AB** plane, and also to QC; PD is perpendicular to the 3-flat of A, B, C, and is therefore parallel to **W**.

$$\mathrm{OM} = \mathrm{OQ}\,\frac{\mathrm{CD}}{\mathrm{CQ}} = q \; vers\; \theta$$

$$\mathrm{MD} = \mathrm{OC}\,\frac{\mathrm{QD}}{\mathrm{QC}} = c \cos \theta$$

$$\mathrm{DP} = \mathrm{PQ} \sin \theta = n_3 \sin \theta = \frac{w}{v} \sin \theta.$$

As vectors

$$\mathrm{OP} = \mathrm{OM} + \mathrm{MD} + \mathrm{DP},$$

$$\therefore\; \overline{\boldsymbol{ab}}^{n}\,\mathbf{C} = \mathbf{Q}_3 \; vers\; \theta + \mathbf{C} \cos \theta + \frac{\mathbf{W}}{v} \sin \theta$$

$$\therefore\; \overline{\mathbf{AB}}^{n}\,\mathbf{C} = a^n b^n \Big[ \big\{ \mathbf{A}\,(b^2\,\mathrm{S}_{ac} - \mathrm{S}_{ab}\,\mathrm{S}_{bc}) + \mathbf{B}(a^2\,\mathrm{S}_{bc} - \mathrm{S}_{ab}\,\mathrm{S}_{ac}) \big\} \frac{vers\;\theta}{v^2}$$
$$+ \mathbf{C} \cos \theta + \frac{\mathbf{W}}{v} \sin \theta \Big].$$

**44.** If **C** is perpendicular to **A** and **B**, then

$$\overline{\mathbf{AB}}^{n}\,\mathbf{C} = a^n b^n \Big\{ \mathbf{C} \cos \theta + \frac{\mathbf{W}}{v} \sin \theta \Big\},$$

and

$$\overline{\boldsymbol{ab}}\,\mathbf{C} = \frac{\mathbf{W}}{v}.$$

$$\therefore\; \overline{\mathbf{AB}}^{n}\,\mathbf{C} = a^n b^n (\cos \theta + \overline{\boldsymbol{ab}} \sin \theta)\,\mathbf{C}$$

$$= a^n b^n \epsilon^{\overline{\boldsymbol{ab}}\theta}\,\mathbf{C}.$$

The rotor $e^{\overline{\boldsymbol{ab}}\theta}$ resembles the rotor $e^{\boldsymbol{a}\theta}$ found in 3-space multiplication. It is evident that similar rotors (quaternions) will be found in all higher space forms.

**45.** The intersections of loci are found as in §32.

EXAMPLE 1. Find the intersection of the 3-flat

$$lll\,(a\boldsymbol{x} + b\boldsymbol{y} + c\boldsymbol{z})$$

with the helix

$$l\,\{\overline{\boldsymbol{xy}}^{n}(\boldsymbol{x} + \boldsymbol{y} + \boldsymbol{z}) + n\boldsymbol{x}\} = l(\boldsymbol{x} + \boldsymbol{y} + \boldsymbol{z} \cos \theta + \boldsymbol{u} \sin \theta + n\boldsymbol{x}).$$

Equating coefficients of $\boldsymbol{x}$, $\boldsymbol{y}$, $\boldsymbol{z}$, $\boldsymbol{u}$,

$$a = n + 1$$
$$b = 1$$
$$c = \cos \theta$$
$$0 = \sin \theta.$$
$$\therefore\; c = \pm 1$$
$$\text{and } n = 0, \pm 2, \pm 4, \text{ etc.}$$

The intersection is

$$l^{n}(n+1)\,\mathbf{x}+\mathbf{y}\pm\mathbf{z},$$

representing two rows of points parallel to X.

EXAMPLE 2.—Find the intersection of the plane $ll\,(a\mathbf{x}+b\mathbf{y})$ with the solid cylinder

$$ {}_{0}l^{4}\,{}_{0}l^{c}\,{}_{0}l^{m}\left\{\overline{\mathbf{x}\mathbf{y}}^{n}\,\mathbf{C}+m\,(\mathbf{x}+\mathbf{u})\right\}=lll\big[c_x\mathbf{x}+c_y\mathbf{y}+(c_z\cos\theta-c_u\sin\theta)\mathbf{z} $$
$$ +\,(c_u\cos\theta+c_z\sin\theta)\,\mathbf{u}\big]. $$

Equating coefficients of $\mathbf{x}$, $\mathbf{y}$,

$$a=c_x+m$$
$$b=c_y,$$

and the plane locus becomes the rectangle

$${}_{0}l^{c}\,{}_{0}l^{m}\left\{(c_x+m)\,\mathbf{x}+c_y\mathbf{y}\right\}.$$

46. PROJECTIONS. To express any vector $\mathbf{K}$ in terms of any four vectors $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, $\mathbf{D}$, not in one 3-flat, write

$$l\mathbf{A}+m\mathbf{B}+n\mathbf{C}+r\mathbf{D}=\mathbf{K}.$$

Then

$$la_x+mb_x+nc_x+rd_x=k_x$$
$$la_y+mb_y+nc_y+rd_y=k_y$$
$$la_z+mb_z+nc_z+rd_z=k_z$$
$$la_u+mb_u+nc_u+rd_u=k_u$$

$$\therefore\quad l=\frac{|\,k_x\,b_y\,c_z\,d_u\,|}{|\,a_x\,b_y\,c_z\,d_u\,|},\qquad m=\frac{|\,a_x\,k_y\,c_z\,d_u\,|}{|\,a_x\,b_y\,c_z\,d_u\,|},$$

$$n=\frac{|\,a_x\,b_y\,k_z\,d_u\,|}{|\,a_x\,b_y\,c_z\,d_u\,|},\qquad r=\frac{|\,a_x\,b_y\,c_z\,k_u\,|}{|\,a_x\,b_y\,c_z\,d_u\,|}.$$

Writing either $l$, $m$, $n$ or $r$ equal to zero the remaining terms of $\mathbf{K}$ are the projection of $\mathbf{K}$ made parallel to the vanishing vector and upon the 3-flat of the remaining vectors. To project $\mathbf{K}$ normally upon the 3-flat of $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, write $\mathbf{D}=\mathbf{W}_{abc}$, then make $r=0$.

The sum of any two terms of $\mathbf{K}$ is the projection of $\mathbf{K}$ upon their plane, made parallel to the plane of the other two vectors.

Loci are projected in the same way.

**47.** As an illustration of the method of the last section we may find the principal orthogonal projections* of the regular 8-cubed tessaract whose edge is unity,

${}_0l^1\,{}_0l^1\,{}_0l^1\,{}_0l^1\,\mathbf{K}$, upon the 3-flats about it.

(1) Parallel to $\boldsymbol{x}$, on the 3-flat of $\boldsymbol{y}$, $\boldsymbol{z}$, $\boldsymbol{u}$, the projection is obtained by writing $k_x = 0$, giving the cube

$${}_0l^1\,{}_0l^1\,{}_0l^1(k_y\,\boldsymbol{y} + k_z\,\boldsymbol{z} + k_u\,\boldsymbol{u}).$$

(2) Parallel to $\boldsymbol{x} + \boldsymbol{y}$. Let $\boldsymbol{x} + \boldsymbol{y} = \mathbf{D}$.

To get three other rectangular vectors we may take

$$\mathbf{A} = \boldsymbol{z}$$
$$\mathbf{B} = \boldsymbol{u}$$
$$\mathbf{C} = \mathbf{W}_{abd} = \boldsymbol{y} - \boldsymbol{x}.$$

Then $l = k_z$, $m = k_u$, $n = \dfrac{k_y - k_x}{2}$.

Writing $r = 0$ the projection becomes

$${}_0l^1\,{}_0l^1\,{}_0l^1\,{}_0l^1\left\{k_z\,\mathbf{A} + k_u\,\mathbf{B} + \frac{k_y - k_x}{2}\mathbf{C}\right\}.$$

And $a = 1$, $b = 1$, $c = \sqrt{2}$.

To express this locus in geometrical terms we note first that since it contains three vectors, not coplanar, with independent variable coefficients, it is a 3-space solid; and in the second place that the original axes, $\boldsymbol{z}$, $\boldsymbol{u}$, which are perpendicular to the line of projection, remain unchanged. The axes $\boldsymbol{x}$, $\boldsymbol{y}$, are each foreshortened in the ratio of $\sqrt{2} : 1$. Projecting $\boldsymbol{x}$ and $\boldsymbol{y}$ by the same plan as for $\mathbf{K}$ we get for the projections

$$\boldsymbol{x}^1 = \tfrac{1}{2}\,\mathbf{C}$$
$$\boldsymbol{y}^1 = -\tfrac{1}{2}\,\mathbf{C}$$

making the total distance $\sqrt{2}$ along $\mathbf{C}$.

Consider next the variables in the locus. $k_z$ and $k_u$ are entirely independent, with limits from 0 to 1, and ${}_0l^1\,{}_0l^1(k_z\,\mathbf{A} + k_u\,\mathbf{B})$ is a square in the $\mathbf{A}\,\mathbf{B}$ plane. The solid is a right square prism whose extension along $\mathbf{C}$ is given by the last term $\dfrac{k_y - k_x}{2}\mathbf{C}$ of the locus. $k_y$ and $k_x$ vary independently from

*For a purely geometric investigation of these projections see the AMERICAN JOURNAL OF MATHEMATICS, Volume XV, No. 2, pages 179-189.

0 to 1. The lower limit of the term occurs when $k_y = 0$, $k_z = 1$, namely, $-\frac{1}{2}\mathbf{C}$; and the upper limit is $\frac{1}{2}\mathbf{C}$. Since $c = \sqrt{2}$, the length of the prism is $\sqrt{2}$ along $\mathbf{C}$.

(3) Parallel to $\boldsymbol{x} + \boldsymbol{y} + \boldsymbol{z}$, $(= \mathbf{D})$.

Take for the other rectangular axes

$$\mathbf{A} = \boldsymbol{u}$$

$$\mathbf{B} = \boldsymbol{x} - \boldsymbol{y}$$

$$\mathbf{C} = \mathbf{W}_{abd} = \boldsymbol{x} + \boldsymbol{y} - 2\boldsymbol{z}.$$

Then $l = k_u$, $m = \dfrac{k_x - k_y}{2}$, $n = \dfrac{k_x + k_y - 2k_z}{6}$.

Put $r = 0$; the projection is

$$\int_0^1\int_0^1\int_0^1\int_0^1 \left\{ k_u\mathbf{A} + \frac{k_x - k_y}{2}\mathbf{B} + \frac{k_x + k_y - 2k_z}{6}\mathbf{C} \right\},$$

where $a = 1$, $b = \sqrt{2}$, $c = \sqrt{6}$.

The figure is again a 3-solid; the axis $\boldsymbol{u}$, perpendicular to $\mathbf{D}$, remaining unchanged. Projecting the other three axes we get

$$\boldsymbol{x}^1 = \tfrac{1}{2}\mathbf{B} + \tfrac{1}{6}\mathbf{C}$$

$$\boldsymbol{y}^1 = -\tfrac{1}{2}\mathbf{B} + \tfrac{1}{6}\mathbf{C}$$

$$\boldsymbol{z}^1 = -\tfrac{1}{3}\mathbf{C}.$$

The length of each of these is $\dfrac{\sqrt{6}}{3}$. This length may be found directly by the equation

$$\sin^2 \boldsymbol{x}\mathbf{D} = 1 - \cos^2 \boldsymbol{x}\mathbf{D} = 1 - \frac{S^2_{xd}}{d^2} = \tfrac{2}{3}.$$

The variable $k_u$ is independent. The figure is therefore a right prism of unit length along $\mathbf{A}$. To find the prism base, or section in the $\mathbf{BC}$ plane, draw the axes $\frac{1}{2}\mathbf{B}$, $\frac{1}{3}\mathbf{C}$, and plot the figure.

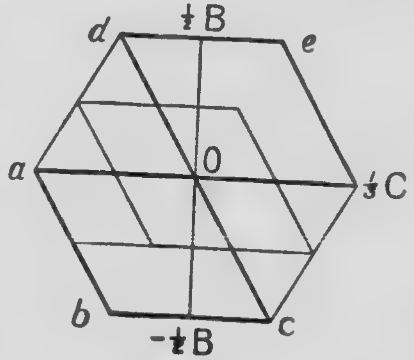


FIG. 14

First, let $k_x = k_y = 0$, while $k_z$ varies from 0 to 1, tracing the line along $\mathbf{C}$ from $-\frac{1}{3}$ to 0, the line $a$O. Next let $k_x = 1$; the locus of $k_z$ is then the line $bc$ from

$$-\frac{\mathbf{B}}{2} - \frac{\mathbf{C}}{6} \text{ to } -\frac{\mathbf{B}}{2} + \frac{\mathbf{C}}{6}.$$

Intermediate values of $k_y$ fill out the parallelogram $ac$.

Next let $k_x = 1$, and proceed as before, obtaining the parallelogram $dC$, whose limiting lines are $de$ from $\frac{\mathsf{B}}{2} - \frac{\mathsf{C}}{6}$ to $\frac{\mathsf{B}}{2} + \frac{\mathsf{C}}{6}$, a[illegible] [illegible] from O to $\frac{\mathsf{C}}{3}$.

Intermediate values of $k_x$ give similar parallelograms commencing at every point along $ad$ and covering the regular hexagon $ace$. The whole projection is a right hexagonal prism. The projected axes $x^1$, $y^1$, $z^1$, are $Oe$, $Oc$, $Oa$.

(4) Parallel to $x + y + z + u$, $(= \mathsf{D})$.

Take for the other rectangular axes

$$\mathsf{A} = x - y$$

$$\mathsf{B} = z - u$$

$$\mathsf{C} = \tfrac{1}{2}\,\mathsf{W}_{abd} = x + y - z - u.$$

Then $l = \frac{k_x - k_y}{2}$, $m = \frac{k_z - k_u}{2}$, $n = \frac{k_x + k_y - k_z - k_u}{4}$,

and the locus of the projection is

$$\int_0^1\int_0^1\int_0^1\int_0^1 \left\{ \frac{k_x - k_y}{2}\mathsf{A} + \frac{k_z - k_u}{2}\mathsf{B} + \frac{k_x + k_y - k_z - k_u}{4}\mathsf{C} \right\},$$

where $a = b = \sqrt{2}$, $c = 2$.

Projecting the axes $x$, $y$, $z$, $u$, we get

$$x^1 = \tfrac{1}{2}\mathsf{A} + \tfrac{1}{4}\mathsf{C}$$

$$y^1 = -\tfrac{1}{2}\mathsf{A} + \tfrac{1}{4}\mathsf{C}$$

$$z^1 = \tfrac{1}{2}\mathsf{B} - \tfrac{1}{4}\mathsf{C}$$

$$u^1 = -\tfrac{1}{2}\mathsf{B} - \tfrac{1}{4}\mathsf{C}$$

and the length of each projected axis is $\frac{\sqrt{3}}{2}$.

To obtain the geometric form of the projection, give to all the variables the value zero, then to each one separately give all values up to unity. This gives four lines from O, identical with the projected axes. With three of these lines as adjacent edges form a parallelopiped, and form three more parallelopipeds with the three other possible groups of the four lines. The sum of these four solids, a rhombic dodekahedron, is the projection required.

# A Geometrical Vector Algebra

## CONTENTS

# Scientific Theology

By T. Proctor Hall, M.A., Ph.D., M.D.

All men learn by experience. From accumulated experiences philosophers draw conclusions valuable to all. Experience being very common and reasoning rather rare in the early ages of humanity, reasoning was highly esteemed, then over-rated, valued as an end and not as a means to wisdom; and so the extravagant pretensions of philosophers fell later into contempt.

Then Science was born. Conclusions were logically drawn from wide and accurately observed experiences. These conclusions were further tested by application to new sets of facts, and corrected as often as found defective. Astrology then gave way to astronomy. The "Black Art" became chemistry. Soothsaying and witchcraft were replaced by psychology.

The old philosophical views depended for their acceptance upon the authority of some great name. The greatest and wisest of men, the men nearest to nature's heart, felt and dimly saw the deep realities of the universe. Some of their immediate followers were able to see the same verities when pointed out to them. The others who accepted their views did so on faith, believing in the ability and truthfulness of their leaders who claimed to see what was beyond their own limited powers.

Scientific conclusions do not rest upon authority, but upon facts, the investigation of which is open to every one, and upon inductive reasoning which is worthless if it quails before the most searching criticism. The pursuit of science is the pursuit of truth inductively.

When we say that theology is unscientific, we do not imply that it is false. We mean that its utterances are given under the authority of great names, and acceptance is asked on the basis of authority rather than of facts and logic. The range of facts required for the construction of a scientific theology is so extensive that theology has been compelled to wait for the fuller development of science. Science has now covered the fields of space, matter and energy, and is rapidly including all organic life. That is to say, it is now generally admitted that in these departments the scientific method of investigation is the only one that gives results worth having.

Because of the complexity of their phenomena, religious experiences are among the last to be scientifically studied. It was necessary that scientists should first become familiar with the simpler phenomena of matter and energy before they were capable of understanding the more abstruse and complex. As the investigation of religious phenomena proceeds there will be developed from it the explanation of the facts, which will form the science of religion. A beginning has been made by William James and a few others, but it is not too much to say that at present no such science exists.

Theology, however, is not limited to religion. It attempts the largest generalizations of which we can conceive, and as a branch of philosophy it can become scientific only as an induction from all the classified facts of experience, of which religious experience is only a small part. At present it is therefore possible to give only a rude outline of scientific theology. Its fuller development waits the growth of biology, and especially of the science of religion.

What follows is written from the standpoint of realism. If the reader be accustomed to think in terms of idealism he may find it necessary to change the terminology.* But the facts presented and the conclusions drawn are true, whatever view is taken as to the ultimate nature of the things considered.

From the unity of the universe Herbert Spencer drew the conclusion that polytheism is untenable. There cannot be more than one ruler of the universe. This conclusion meets with the approval of all scientists. Whether there is a God at all is a question for further investigation.

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*The succeeding argument may be outlined as follows in terms of idealism:—I exist. I have sensations, by the aid of which I form concepts. Men are concepts of my mind. Some unknown power has produced in me concepts of men, and also a concept of me which differs from my other concepts of men no more than they differ from each other. But I clearly distinguish me, the ego which has these concepts, from my concept of me. Since there is a "me" causatively related to my concept of me, I infer the existence of men causatively related to my concepts of men.

Other concepts I have; such as of material things, from which I infer the existence of matter; of force and motion, from which I infer energy; of form and relation, from which I infer space; of speed and duration, from which I infer time. And I distinguish these four things, time, space, matter, energy, whose existence I have inferred, from all other things; because I am compelled to infer that they are simple in their nature and unchangeable in quantity. The are the necessary and permanent bases of all my experience, and I name them *cosmic elements*. For reasons stated in the text I include "spirit" among the cosmic elements.

The ultimate source of experience, which I call God, has in it at least all that is common to these cosmic elements.

Let us first review some fundamental generalizations of science.

*Time* is something measurable. It enters into many other quantities, such as speed and power. But it is unchangeable, uncreatable and indestructible by any known power. It is as impossible for any of us to march one moment ahead into the future as it is to fall back into the past. We are in the march to stay, as long as we are in the world. Time is for us a cosmic element.

*Space* is a second cosmic element. It is uncreatable and indestructible. It is like itself only, and cannot be confused with anything else in the universe.

*Matter* has many forms. For more than a century the elements withstood the efforts of chemists to change them one into another. Nevertheless every scientist has believed that though the elements are many they are all forms of one "matter," whose properties are described in every text-book of physics. The transformations of the radio-active elements recently discovered are of intense interest, but they add nothing, because nothing could be added, to the belief of the scientist in the essential oneness of matter, of which he sees only the varying forms. Matter, including under this term the ether, is indestructible and uncreatable by any known power. It is a third cosmic element.

One other such element is known to science. *Energy* (as defined in physics) has existence in many convertible forms, as motion, strain, heat, light, sound and electricity. But, though easily transformed, energy is indestructible and uncreatable. That is to say, not the least particle of energy is ever known to be created or destroyed. Energy is a fourth cosmic element.

These four are the only cosmic elements known to physical science. All other things in the visible world are variable in quantity as well as form. These have also the common character of simplicity. The smallest conceivable part of any of them does not differ in kind from any other part of the same element. All other things are complex and can be so divided that some of the parts are essentially unlike other parts.

These four elements and their combinations form the whole physical universe. Outside of them we know only one other class of things, namely, those directly relating to conscious life. In this class we know of no single thing that has the marks of a cosmic element, nothing that remains constant in quantity through all changes of quality and form. But by analogy we may infer that there is to the things of this class a substratum which does remain constant. Let this substratum be named

*"Spirit."* Then the argument for the existence of spirit as a fifth cosmic element is as follows: Five distinct classes of things—time, space, matter, energy, and the things relating to conscious life—form with their combinations the known universe. The first four of these are each decided by scientists to be permanent in quantity. The fifth class must also be permanent in quantity or else subject to partial annihilation and creation from day to day—an unthinkable condition. If the fifth class is quantitatively permanent it must have under its apparently variable phenomena a common and permanent substratum (here named "spirit") having the characters of a cosmic element.

If this argument seems inconclusive, let it pass. It may be that these cosmic elements are not absolutely permanent. It may be that, just as has been found with some of the elements of matter, there is a slow transformation taking place, one cosmic element changing into another. It may be that spirit is the least constant of the five, and that the evolution of the ages is making spirit a larger and larger element of the universe. It may be that other cosmic elements exist, of which we have not yet the faintest intimation. These are speculations going far beyond our knowledge.

Let us return to things we know. About the ultimate nature of the chemical elements we know a little. The little that we know about the transformation of matter from one element into another does not invalidate any of our previous knowledge of the elements. We continue to call them elements, and we know that they mark a definite stage of aggregation of matter. So also any speculations we may indulge in about the ultimate nature of the cosmic elements cannot alter the fact that they are a very distinct and definite stage in the formation of the universe, and no future discoveries can alter that fact.

If five or more cosmic elements form the universe, there must be some fundamental bond amongst them, something holding them together as a universe, some substratum common to them all and of which each is a manifestation, just as silver and carbon are two manifestations or forms of matter, and as heat and electricity are two manifestations of energy. To this common substratum let us apply the term "God."

It may be objected that the word "God" is already in use with an entirely different meaning and therefore should not be used here. But if it can be shown that the word as here defined is similar in its essential meaning to the term in common use, the objection falls. The only knowledge we can possibly have of God (using the word in the theological sense) is gleaned from his expression in the universe. But as long

as the term is philosophically defined there will continue to be as many gods as there are thinkers to define it. Moreover the use of any other term for this substratum, this common bond of the cosmic elements, would suggest to the reader the hypothesis of two gods, one immanent, belonging to the whole universe, the other an outsider, a Royal Mechanic who impresses his will upon the universe. Though the latter view of God is commonly taken by ignorant men, it has not the sanction of the greatest theologians. It would be folly to claim that all the efforts of the ages to find out God have been wasted. Men have blundered and will blunder, but there has been progress. If science has now reached a stage where it has some contribution of value to offer in the search after God, let us accept it and if necessary modify our ideas; but let us not forget what has been already learned, and that we can at best bring only a small contribution to this age-long quest. The God discerned by science is not a new God, but the same Eternal seen from another standpoint and seen in some respects more clearly.

How do we learn the properties of matter? We know matter only in its various forms or manifestations, as iron, wood, coal, rock, water, air, etc. Whatever properties are present to some degree in every form of matter are properties of matter, such as mass, elasticity, volume. Properties found only in a few forms of matter are not considered fundamental. Magnetism belongs to iron, but not to copper; it is therefore not a property of matter. So also we know energy by finding what properties are common to its various forms. Proceeding in the same way to study the cosmic elements, we may gain a fuller knowledge of their substratum, God. Taking in each cosmic element one prominent charter that is also found to some degree in each of the others, and is therefore universal, we may get, in rude outline, the attributes of God. Many others may be added, and every addition to our knowledge of the universe increases the possibilities of our knowledge of God.

From *Time* we get the suggestion that God is eternal; from *Space*, infinite; from *Matter*, that all action is according to regular law; from Energy, the principle of evolution; from *Spirit*, intelligence. The God of the universe is thus an eternal, infinite, consistent, evolving, intelligent God. These are attributes to which every scientist must give immediate assent. Careful study will enable him to add many more. Every one is at liberty to investigate and draw his own conclusions, as in every branch of science. The dogmatist is he who draws conclusions without sufficient investigation.

Men's ideas of God have been fragmentary and distorted. According to theologians God is spirit. But this is evidently no more

true than the statement of the astronomer that God is infinity, or of the evolutionist that God is power, or of the materialist that God is matter. Each of these statements is an approximation toward truth. Since men are characteristically and essentially spirit, their first clear ideas of God were as spirit. Knowing little of matter, they clothed God with the common attributes of men; described him as irregular, capricious, selfish, vengeful, tyrannical; so that Ingersoll was moved to say, "An honest God is the noblest work of man." With the development of natural science came the knowledge of natural law and evolution. Scientists could no longer accept the theologians' ideas of God. By a revulsion of thought many men turned from the theological idea to an equally fragmentary idea based on materialism or on evolution. Caprice cannot coexist with natural law, and absolute and final perfectness is inconsistent with evolution. Gradually it has come to be generally realized that law is universal, and the theological idea of God has been modified accordingly. Evolution as a process in nature is now generally accepted, but it has not yet found a place in the theological system. The idea of an omnipotent and absolutely perfect God cannot be reconciled with the imperfections of the universe as we know it. But an evolving God is in harmony with all the facts we know, and cuts the knot of many an unsolved problem. This idea of evolution as an essential character of God is not easy to grasp fully. It contradicts the common view of God as unchangeable or as the Absolute. The whole known universe is in the march of evolution. Its essence is therefore evolving. Its plan is being formed. Its aims are more clearly defined now than ever before, its consciousness clearer and more extensive. Of a beginning or an end science knows nothing. The process only is seen. We are conscious portions of the universe. God is in us and is through us enlarging the plan and developing its parts. We have no reason to suppose there was ever an original design perfect in all details. The design itself is growing into consciousness in the heart of the universe, just as it grows in the life of an individual.

The problem of evil is recognized and its solution attempted by every known system of theology, from the ancient Egyptian down to the most modern Eddyism. The Zoroastrian doctrine of twin deities —one of them good, the creator of light and life; and the other evil, the creator of darkness and death—has been partly incorporated into the Miltonian doctrine of a good and powerful God who has already defeated and will ultimately destroy the arch-rebel Satan, the source of all evil. Mrs. Eddy shifts evil into the realm of idealism, explaining it as delusions of mortal mind,—which are to be at last all cleared up.

Every theological system recognizes that a contest of some sort is going on, and that evil is being overcome by good. But the origin of the fight, or the necessity for it, is nowhere made clear. If the creator of life cannot overcome his evil twin, or cannot destroy Satan, or cannot prevent "delusions of mortal mind," these causes of evil, he is not omnipotent. If he can but will not, he is not good.

Granting evolution as a fundamental principle in the universe, this conflict takes on a different appearance. Intelligence being also fundamental, there appear before us always ideals, which are the scouts of the evolutionary process. Present conditions are "good" as they make for our ideals, and "bad" when they do not. To a democrat the growth of the trusts was only bad,—his ideal was in the past. To a socialist the same industrial movement appeared good because it pointed toward his ideal of a co-operative future. The general trend of ideals, themselves the product of evolution, is necessarily in harmony with the general trend of evolution, of events, and hence it has come to be an essential part of doctrine in every religion that good will finally conquer. Social conditions that are now universally condemned are called bad because ideals are already picturing out the coming advance into something better. And when these ideals have become realities, higher ideals will take their place, compelling a re-classification of the events and conditions to suit the new ideals. So the forces of the universe mould its materials by means of intelligence into forms of higher organization. And as the universe evolves, as knowledge becomes broader and truer, as ideals become more cosmic and events follow more closely, more and more of the universe is classed as "good," and stronger and fuller appears the harmony of all.

> "Yet I doubt not through the ages one increasing purpose runs,
> And the thoughts of men are widened with the process of the suns."

As an example of further investigation into the nature of God, let us consider the question whether or not God is love. This is equivalent to the question, Does love or its analogue appear in every part of the universe where such appearance is possible?

Love is not in any sense known to belong to time or space *per se*. Neither is its opposite. Nor can we conceive of either love or hate belonging to what we know of time and space. These two cosmic elements may therefore stand aside from this question. Taking the next element, matter, we find an analogue of love in gravitation, a universal attraction. If there are any atoms that repel each other, they do not belong to the visible universe, for they would ages ago have made their

way toward the boundary of the ether, into the outer darkness, hence beyond the possibilities of our knowledge. In all atoms we find also chemical attraction more or less strongly developed. Cases of apparent repulsion are probably apparent only, and are easily explained on other grounds. Coming to the next cosmic element, energy, we find here and there antagonistic forms. For example two opposite electric charges tend to coalesce, disappear as electricity, and become light and heat. With such small and insignificant apparent exceptions, the energy of the universe is harmonious. In the fifth cosmic element, spirit, both love and hate appear. It remains to be seen whether both are real or whether one is merely the relative absence of the other. As a factor in life, love is supreme over hate, else the human race would perish. Every man loves himself; and does what he thinks right and best for himself—perhaps not by your standards, but by his own. At first, in early childhood, his "self" includes only his own wishes and feelings. Later as he develops it comes to include successively his own body, his possessions, his family, his friends, his clan, his society, his class, his nation, his race, and all life. Everything outside his "self," that threatens to interfere with it, rouses antagonism because of his devotion to his "self." Here love is clearly seen to be the prime moving force, which appears as hate only in certain undeveloped conditions. Hate is then only limitation and negation. By definition, God is the essence of the whole universe; his love then extends to all that of which he is the essence, leaving no place for hate.

As a second example consider the question, "Is God just?" By justice is meant equality of conditions and opportunities; or equal results for equal efforts of different individuals. As so defined justice is conspicuous by its absence from the universe. Paul claims that one vessel is made for honor, another for dishonor; and the inequality of conditions among men is evident. In the field of energy a very slight factor often makes the difference between intense action and almost no action. In matter, too, there is endless diversity of relation. And no two portions of either space or time are similarly related to the rest of the universe. Hence justice does not belong to the essence of the universe.

Justice is, after all, only a kind of rule-of-thumb that we apply to human affairs in default of fuller knowledge and stronger love. It is a negative standard, exceedingly defective, and wholly inferior to intelligent love. It could not be fundamental in the universe, and God is infinitely superior to it. In the evolutionary sense, God is not wholly good, but is becoming good; and he is not limited to justice because he has already gone far beyond it in unlimited love.

Whatever may be thought of the illustrations just given, the main contention of this paper is that it is now possible to apply the scientific method to investigate the characters of God. The facts of religious experience will ultimately be a great aid in such investigation, but as yet they have not been sufficiently studied. This method of investigation, being the most reliable and accurate known, must supersede all others and give later a scientific theology in which men of all creeds and races will agree.

In conclusion, the method of investigation here outlined and illustrated leads toward the center and source of being, which center is generally designated as God. The use of any other term instead of "God" would lead to wrong inferences. Outside of this center there is no known God, and he is known only through study of the universe.

# Marine Biology in British Columbia

By C. McLean Fraser, Ph.D.,
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(Read Feb. 19, 1913.)

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The problem of Marine Biology in British Columbia is a very extensive one, so much so that it is entirely impossible to give any adequate idea of its scope in a brief paper such as this. There are 7,000 miles of coast in this province, while if the inlets are included, together with the shores of the islands, there is an estimated tide-water line of 27,000 miles. Even the casual observer, who sees the crabs among the rocks and the shellfish in the sand, must associate the idea of such extent of coast line with that of a wonderful fullness of life.

Nor can the shore and shallow water claim a monopoly of all that is of interest. In the territorial waters alone, a depth of 200 fathoms or over is reached in many places. The possibility of bathymetrical variation is thus as great as that due to geographical distribution.

The province affords the richest hunting ground for rare and valuable game in the temperate regions of the world today, and the same is probably true, in an economical as well as in a biological sense, of the British Columbia waters. The Japan Current, the most beneficent of caterers, gathers the choicest morsels from the whole of the North Pacific to furnish the food supply for the inhabitants of these waters, hence the wealth, the prodigality, in variety and in quantity, shown already in the case of the food fishes, but just as true of a host of other forms.

With all this extent and with all this richness, what has been done towards taking advantage of the wealth? Very little. Not that those who have worked have not done their work well, but that there have been so few laborers for so vast a field. The pioneers have hewed out a little clearing and have gathered a harvest of knowledge. The richness of the region is becoming recognized, and already the forerunners of large forces are coming or are here.

Who were these pioneers and what is the value of the work they have done? In any field that is new geographically, if biology is to obtain a permanent foothold, the taxonomist must first be called upon to classify the commonest forms, so that the classes, orders, genera, species, most abundantly represented, may be made known, after which investigators in other branches of the science, knowing what material is

available, may carry out research work in their own particular lines. But here a great difficulty comes in. So many systematic papers have been published for each class, and even in some cases for each order, of plants and animals, that only those who have made an intensive study of special classes, are at all likely to have literature complete enough to do satisfactory work in classification in that field. What a hopeless task, therefore, it is to undertake to get together the literature necessary for the classification of all marine life. In very few of the world's libraries is there any pretence of doing anything of the kind. Even in many of the largest scientific libraries on this continent there is not literature enough to make it possible to attack successfully even a single class. An investigator in a new field, with only his own library to consult, has little chance to give exhaustive treatment to the classification of a great variety of forms.

How is the difficulty overcome? Only in one practical way, which is by collecting specimens and sending them to authorities for identification. This is what some of the pioneers in B. C. biology did; and in so doing they became familiar with the names, general appearance and something of the life history of such a host of forms that it seems a marvel how their knowledge was obtained and retained. These men are of the type often spoken of as the "old time naturalist." Unfortunately there are too few of them at the present day. Some of them had to depend largely upon their own unaided efforts; but others were supported by such institutions as the Geological Survey and the Natural History Society. Chief among them may be mentioned Dawson, Richardson, Lord, Lyall, Taylor, Newcombe and Macoun. To all of these and to their assistants, the biologist of the present day owes much. All of them are too well known to require any other eulogy than the mention of their names.

In the last decade, the good work started by the pioneers mentioned has been continued under circumstances more favorable than those under which they labored. In 1901, the Minnesota Seaside Station was established at Port Renfrew, on the west coast of Vancouver Island, largely through the instrumentality of Miss Tilden, who, in years previous, had done much alga collecting in the Vancouver Island and Puget Sound regions. Much good work, especially in marine botany, was done at that station. But later Miss Tilden's interest was transferred to the algæ of the South Seas; and no regular work has been attempted since 1907. At about the same time as the Minnesota Station was established, Professor Kincaid of the University of Washington began exploring among the San Juan Islands. At first

this was done largely on his own responsibility; but later the interest of the university authorities was aroused to such an extent that a fine station was established on San Juan Island near Friday Harbor. The interest continued to grow. The co-operation of the biological departments of other universities and colleges has made the expansion very noticeable, until now the institution is largely attended during the summer months and much good work is being done. This is not a B. C. station; but it is so near the boundary that any information obtained from material from the San Juan waters is as useful to us in British Columbia as it is to the State of Washington, and no review of the work done in these waters would be complete without mention of Professor Kincaid, who is one of the most omnivorous collectors of the present day. Five years ago the Dominion Station was established at Departure Bay, near Nanaimo, largely on account of the representations made by the late Mr. Taylor, the first president of this Academy. Since its establishment it has been adding its quota to the biological knowledge of the region. The facilities for carrying on investigations are improving year by year; but more men are needed to make use of the facilities that are afforded. Finally the Provincial Department of Marine and Fisheries has inaugurated a policy of engaging men to carry on research along lines that are largely economic but none the less scientific. Gilbert and Thompson have started in already; and I am credibly informed that provision will be made for others to follow. Besides all of these, the various officers of the Marine and Fisheries Departments, federal and provincial, in connection with their regular duties, have added much to the knowledge of the life history, habits, etc., of many inhabitants of the sea. We know little about their work unless we consult the blue books; but it is of none the less value on that account. The more co-operation there is between men of this class and others engaged in biological work, the greater will be the benefit to both and to the province at large.

While due credit must be given to those who have spent so much time in becoming acquainted with general conditions, it must not be forgotten that much of their knowledge could not well have been correlated without the assistance of the specialists in taxonomy, who gave of their time and their energy to classify material sent them, with little to remunerate them but the love for their work. These should certainly have mention in a review of this nature. A complete bibliography covering the whole field would give the best idea of what work has been done; but that cannot be given in the present paper. It is possible only to mention in a general way something of what has been accomplished. While some biologists, like Whiteaves, for example,

have done systematic work in several classes, the work of each individual has been usually confined to one class, or at most, to one sub-kingdom. Consequently it seems best to consider the sub-kingdoms individually to cover the ground in the shortest time.

I shall begin with the investigators in marine botany. In this the most important work has been done in connection with the Minnesota Station, by MacMillan, Tilden and assistants. Within the last five years Macoun has collected much. Some of this he classified himself, but much of it he sent to Collins for diagnosis. Almost all this work has been done on the algæ. As far as I am aware, no work has been published on the Protophyta.

In the animal kingdom, the same may be said of the Protozoa as of the Protophyta, although a description of some local species is likely to be found in Cushman's "North Pacific Foraminifera." Many of the Protozoa occur in the plankton, which has received some incidental treatment, but no serious consideration. Other species may be found attached to Metazoan species or to seaweed, and hence may have been observed often enough, but only with a passing interest.

The Porifera or sponges are well represented. On these Lambe has done a great deal of work and has published several papers. The collection made by Macoun and assistants for the Geological Survey is at present with von Lendenfeld, who is preparing a report containing a reference to these species.

The Cœlenterate sub-kingdom is very extensive, and has been worked up by classes. In the early days, Hincks and Allman reported on hydroid material collected by Dawson and Richardson. Later Clark, Hartlaub, Nutting, and Calkins published papers on hydroids from these and contiguous waters. I have done some work on these myself, and hope to have a more complete report ready in the near future. Little has been done on the Medusæ, though some B. C. species find mention in papers by Agassiz, Bigelow and Mayer. McMurrich is working on a collection from Departure Bay at present. He has reported on the Actinians also. Apparently the Alcyonarians are not numerous. A description of most of the species is found in the papers by Nutting and Torrey, though they have not been described from these waters especially. The Ctenophores are pretty well covered in papers by Torrey and Bigelow.

In the large Annelid group there has been little done, although there seems to be a great abundance and variety of material. Potts and

Pixell collected at the Departure Bay station in 1911; but I have seen no published report of the results obtained.

Parker's heterogeneous sub-kingdom, the Molluscoida, has been much neglected. The Bryozoa are very plentiful, but no satisfactory report on them has been prepared. Hincks reported upon some material collected by Dawson in the neighborhood of Queen Charlotte Islands, and Robertson has made mention of some B. C. species in her Pacific Coast papers; but the great mass of them remain untouched. The Brachiopods have received incidental treatment by some of the conchologists, but that is all. Some species of the erratic Phoronid group have been studied by Pixell.

Like the Cœlenterata, the sub-kingdom Echinodermata is very extensive, and is abundantly represented in this region. The Asteroidea or starfish have received a great deal of attention from Verrill, who has an extensive report nearly ready at the present time. Fisher has done and is doing much work in the group, and earlier Whiteaves made some reports. Clark's large work on the "Ophiurians of the North Pacific" includes most of the B. C. species of brittle stars. I know of no papers bearing directly on the B. C. echinoids, which include the sea urchins, cake and heart urchins, although several species are abundant. The same is true of the holothurians or sea-cucumbers; but some of Fisher's papers may include some of the species. The few species of crinoids or sea-lilies are probably all described by Clark.

In the Arthropoda only the Crustacea are marine. Some orders of these have been well looked after; but others have been entirely neglected, the latter including chiefly the smaller forms usually found in the plankton. Smith and Bate have done some general work on the class. Taylor prepared a paper on the Malacostraca for the Marine and Fisheries Report, but it has not appeared in print yet. Rathbun has reported on the Decapods; and the papers by Pilsbury on the Cirripedia or barnacles, and by Richardson on the Isopods, include many B. C. forms.

The Mollusca has attracted more investigators than any other invertebrate group, probably because the region offers so good a field, and because the attractiveness of shells in the gross is more apparent than it is in most of the other groups. The orders in which the shell is not pronounced or is entirely absent, have not received the same attention as the others. The nudibranchs have scarcely been touched, although MacFarland's papers probably cover many of the species. The cephalopods or cuttle-fish have been described by Berry. Among the

conchologists may be included Carpenter, Baird, Lord, Whiteaves, Smith, Taylor, Newcombe, Hanham, Dall and Bartsch.

Coming now to the Chordata, species of Balanoglossus have been reported, but I have not seen any of them described. Huntsman has worked up the ascidians, and Macoun has diagnosed the birds. The marine mammals, with the exception of the whales that have been exploited commercially, have received little attention. Naturally, as the fish are of such economic value, much attention has been paid to them; but yet no systematic survey has been made. Most of the species that have been procured have been taken accidentally, or at best, incidentally, in connection with the fisheries for commercial food fish. Among those who have contributed to the ichthyology of B. C. are: Lord, Bean, Jordan, Gilbert, Gill, Evermann, Snyder, McMurrich and Taylor. Taylor's paper has not yet been published. Many of the Fisheries Reports give valuable information on this group, in connection with life history, distribution and economic value.

This list, incomplete though it is, will indicate the widespread interest that has been set up and maintained on the basis of the work done by a few naturalists who prosecuted their labors with such zeal when encouragement and appreciation were sadly lacking. Although the possibilities for investigation are without limit, a beginning has been made, and many problems appear right to hand, waiting to afford results to any and all who will attack them with real desire for accomplishment. Some of these that most readily suggest themselves may be mentioned, not necessarily in the order of their importance.

A general biological survey is very much needed. Heretofore, owing largely to lack of facilities, observations have been made here and there, at points isolated from one another, without any system or co-operation. If all the forces now at work would come to some agreement in that regard, much might be done in a general way in a short time. If a survey were made it would be possible to find out what special localities would be best suited for carrying on particular investigations, so that the time now lost in wandering around looking for material, might be put to a better use. The distribution of all the common forms would be made known, and many forms of value, hitherto untouched, would be discovered. This would entail more taxonomic work in adding species to the lists in the classes already studied, and in making out lists in classes hitherto neglected. The survey, as far as it has been carried at present, has been largely in connection with littoral forms and those that can be obtained in 30 fathoms or less. The plankton, in its way, offers just as rich a field. That the most of the

species are microscopic does not detract from their importance. The protozoa, protophyta, eggs, larvæ and such small crustaceans as the Copepoda, Ostracoda and Cladocera, are of interest in their morphological relations, and also because they form the food material, directly or indirectly, for the great mass of marine life. Quantitative and distributional research on the plankton should hence be of great value. Of the larger forms, your attention has been called to the classes that have been most neglected. The lists of Bryozoa, Nudibranchs, Annelids, etc., should be made as complete as in the classes that have received the greatest attention; while many others should receive important additions. Even among the fish, the chances are that there are many species, some of them common enough, that are not known. Tide pool and shallow water forms, except in a few localities, have scarcely been touched; and the bottom forms have not received due attention. When the distribution of the common species is known, it will be an easy matter to get material for investigators in other branches of the science: the physiologist, the biochemist, the experimental zoologist and botanist, the morphologist, the comparative anatomist, the embryologist and the ecologist will all find something to do. Enough is known already to start all of them going; but with a complete survey they would have a chance to do much better work.

In the working out of heredity problems, the cœlenterates and the echinoderms have received a great deal of attention. I believe I am correct in saying that nowhere along the shores of this continent at least, can be found greater numbers or a better variety of these two classes than can be found in these waters. The sea-urchin has been a favorite for experiments in fertilization. We have in abundance two species suited for such work, the smaller green urchin, *Strongylocentrotus drobachiensis*, and the large purple one, *S. franciscanus*. Experiments concerning the effect of controlled stimuli have been tried extensively on the starfish. There is scarcely anything more common along the coast than the starfish; and if one species does not prove satisfactory, there is a large enough variety to choose from. The anemone has been used for similar experiments. It would be impossible to find *Metridium* more plentiful than it is on some of the rocks in the Gulf. A species of *Anthopleura* and several others beside are more or less abundant. Co-ordination experiments have been triea on medusæ. *Gonionemus murbachi* has been the great favorite for these at Wood's Hole. *Gonionemus vertens*, which may or may not be the same species, is much more abundant in the Gulf of Georgia than the Atlantic species is at Wood's Hole. Many other genera are well represented. Regeneration experiments have been tried on hydroids; and they have been used

in heredity problems. We may even say that Weissmannism is founded largely on the results obtained from the study of that group. We have them in plenty. I might quote one case in corroboration. In one haul of a two-foot hand dredge, down less than five minutes, in Northumberland Strait, I obtained 36 species, a number more than one-third as great as that which represents all those obtained within a day's trip of Wood's Hole in 30 years. (I mention Wood's Hole because in all probability there has been more work done there than at all the other stations on the continent combined.) Enough has been said to indicate that the experimental zoologist need not remain idle, and on the same basis there are plenty of possibilities for the physiologist and biochemist.

The morphologist and the embryologist will find plenty of scope in working on life history. When it comes down to the facts of the matter, the life history of comparatively few of the myriads of species, here or elsewhere, is known. This is particularly true of marine forms. In the case of those that live continually in the deeper water, it is a very difficult matter to find out much about them; but even the littoral and shallow water forms are very little known. One reason for this is, that owing to the exigencies of climate, it is hard to keep track of even the most conspicuous forms throughout the year. This is the very reason why scarcely any locality in the temperate zones affords so satisfactory an opportunity to study life history as our own. There is nothing to hinder the work being carried on throughout the year; and the species are so numerous that there is no danger of ever running out of a supply, because every species, no matter how much it is like its nearest of kin, has some special feature in its life history that, when known, will help to unravel the great skein of relationship in which every biologist has a deep interest.

The study of life history has a great economic value in some cases. It has been and is very necessary in the case of food fishes. Much work has been done on the various species of salmon, in connection with the hatcheries for the early stages, and by McMurrich and Gilbert for the adults; but much still remains to be done before even an approximately complete knowledge is obtained of the whole life cycle. Other fish have scarcely been studied at all. Even from a dollars and cents point of view, it will surely pay to undertake such work. When last year, with fish value of over thirteen and a half millions, British Columbia easily led the provinces of the Dominion, is it not worth while to find out something as to the life history and habits of at least the principal species that go to produce such a value? Millions of herring are caught

in a single night; but nothing is known of the why and the wherefore of their migrations. If there is any danger of overfishing, should it not be known? If there is no such danger, the best methods of fishing and curing should be introduced. Halibut is brought into market, but little is known concerning it except that it can be caught on certain banks at certain times. This lack of knowledge is even more decided in connection with other flat fish. Now that trawling has a good chance of becoming a large factor in the fishing industry, with the British Columbia Fisheries Company already operating successfully off Skidegate, and the Canada Fish and Storage Company of Prince Rupert with two boats ready to start and more on the way from Grimsby, an excellent chance is now afforded for research in that interesting group. A careful search might reveal further that there are many species of good food fish readily available which at present are receiving no attention whatever.

While speaking of economic value, other cases may well be considered; because the fishes have not a monopoly of commercial value. Everywhere, reaches, tide-flats and beds of sand and gravel, make suitable abiding places for innumerable shellfish. Even at present their commercial value must be taken into account. But that value is as nothing compared with what it might be. Dr. Stafford, after working on the oyster for some time, came to the conclusion that not only could the large eastern oyster thrive in the waters of Vancouver Island, but that it could reproduce in these waters. Many attempts have been made farther south to get them to reproduce, but I believe in no case have the experiments met with success. Oyster farming such as is done in Japan, in some parts of Europe, and even in some parts of the Eastern United States and Canada, is a very profitable venture. There seems no reason why it should not be so here; but further experiments must be made before success as a commercial venture is assured. But the oyster is not the only valuable shellfish. Various clams and scallops are very desirable as food, while cockles, mussels, limpets and other similar forms are considered great delicacies in various parts of the world. The provincial department is going into this matter; it has shellfish jurisdiction now, and has set Mr. Thompson at the problem of distribution and extent of the beds of the edible forms. He has sent in a preliminary report on his work of last summer, and will continue the work next summer. Experiments as to edibility, food value and life history of many of these shellfish would add much to the value of the work now in progress.

What has been said as to the study of shellfish may as truly be said of crustaceans. Many crabs are caught in some localities; but the

marketing of them can scarcely be called an industry here, though there seems no reason why it should not be. If one visits the fish markets of New York or Boston, he gets an idea of what a prominent place the various crabs occupy as food material in the East. Why should it not be the same in Vancouver or Victoria? The shrimp industry is of great importance in the State of Washington. It is possible that a thorough search would reveal just as good shrimp ground in the B. C. waters as in the waters but very little farther south. The species caught for market are certainly found farther north in the Gulf. Are they there in large quantities? As far as appearances go, the conditions seem favorable for the growth of lobsters along many parts of the coast. A feeble attempt was made to introduce them some years ago, but as there were no special arrangements for their care, no one knows whether any of them lived or not. Such experiments as these require strict and continued attention for years before a satisfactory conclusion can be reached.

Another problem that affects the fisherman from a different standpoint may be mentioned. The dogfish is a great nuisance to nets and lines. The dogfish of the Pacific is much like the dogfish of the Atlantic. The biochemist has made it possible to get value out of the Atlantic dogfish: he should be able to get similar value out of the Pacific species. At several points in the Maritime Provinces to the east, reduction works yield profit from oil and fertilizer. There is room for some such in the Maritime Province of the west.

While we have the biochemist, there is another problem, a large one, that we wish him to attack. Owing to the amount of fresh water that is poured into the sea by rivers, great and small, there is much variation in the salinity of the water at various points, all the way from the high percentage of the regular ocean water to the brackish or even fresh water at the outlets of the rivers. In the case of large rivers, the influence of the fresh water may extend a long way. The water of the Fraser crosses the Gulf, on the surface at least, to Gabriola, Valdez and Galiano Islands. How does this variation in salinity affect the forms that come in contact with it? Are there species peculiar to each degree of variation? Are there variations in the same species? Or can any difference be noticed? There may not be much difference in the freely swimming forms, but experience indicates that there may be much difference in sessile or slowly moving forms. Nowhere can greater variety in salinity be found than here, hence nowhere can the problem be studied to greater advantage. It has its economic value also; for a slight difference in flavor in an edible species may make

the difference between commercial failure and success. The problem is not confined to animals. It has an even greater bearing on marine plants, as a greater percentage of these are sessile forms.

Hitherto I have said little directly about plants, but the most of what I have said applies to the flora as well as the fauna. Animal life is dependent on plant life, directly or indirectly, for food material; hence it is just as necessary to obtain a knowledge of the nature, extent, and distribution of plants as of animals. In both cases the relation of each species to its environment and to other species forms a large and important problem in itself.

From an economic standpoint there are two problems at least, that loom up large in connection with marine plants. In some parts of the world, an important industry has been established in obtaining iodine and its compounds from seaweed. The seaweed is in abundance everywhere here. It might be worth while experimenting as to the possibility of obtaining these substances in quantity enough to be of economic value. Experiments have been conducted, with how much success I am not prepared to say, on the Pacific Coast of the United States, with the idea of obtaining potash from kelp. If that can be done profitably, there are large enough beds of *Nereocystis* growing on the reefs every year to provide any amount of raw material.

It is quite true that financial assistance is needed to supply the facilities to cope with these larger economic problems. They need government backing. In these days when every progressive government the world over is becoming more sympathetic towards scientific endeavor, more cognizant of the results of scientific investigations, the federal and provincial governments are sure to give due consideration to any feasible plan to advance such work. It is only reasonable that they should ask for men who will get results before they will give much encouragement. A call that has often been issued must be taken up once more, a call for men, men full of scientific spirit, willing to attack with patience and zeal the manifold problems of so rich a field. Apparently we must depend on university trained men, and there are not enough with interest in such work, available. In Britain, as in many of the older countries, the professional man and the artisan often take up the study of some science as a hobby, and do some very valuable work. In Canada and in the United States, "gross materialism" seems to crush out all desire for any hobby that has not to do with the making or spending of money.

It is the duty as well as the privilege of this Academy, by every means at its command, to foster a desire to have this work done with the greatest possible despatch. Every member, besides being fully informed of the needs of his own particular branch of science, should take a sympathetic interest in all the other branches. Doubtless he does so. In considering marine biology, this discussion may help to give further insight into the possibilities for profitable research. If your interest is aroused, show it by placing the situation fairly before others who by their personal efforts or by their influence, can give valuable assistance, and by giving encouragement to those who are already engaged in the work.

# A Review of British Columbia Geology.

By Edward M. Burwash, M.A., Ph. D.

(Read Feb. 19, 1913.)

The object of this paper is briefly to summarize the development of geological knowledge with regard to British Columbia, outline its present condition, and suggest some of the problems that remain to be solved.

Since the geological study of any area presupposes its geographical exploration and the existence of correct maps, it may truthfully be said that the beginnings of our present knowledge were made by the earliest explorers of the province. None of these did any work of permanent value until the advent of Vancouver, whose maps of parts of the coast are remarkably accurate. The work of Mackenzie and Fraser prepared the way for others, whose work was of a more detailed and accurate sort than that of the pioneers could hope to be. The knowledge gained by the fur traders during the early part of the last century, and by the early miners at a somewhat later date, was not of a scientific order, and for the most part was never committed to paper, but served its purpose in the oral state by expediting the work of the later explorers.

Contributions of a definitely scientific nature began to make their appearance about the middle of the last century. The earliest are perhaps those referred to by Dr. Hector, in a paper read before the Geological Society of London in April, 1861, as follows:—*"Some fossils transmitted to the Jermyn Street Museum many years ago from Vancouver Island were first rightly recognized by the late Professor E. Forbes as being cretaceous, but so far as the writer has been able to ascertain, no statement to that effect was ever published by Forbes. Mr. F. B. Meek appears to have been the first palæontologist to publish specific descriptions of rocks from this part of Vancouver Island. His earliest paper on the subject appeared in 1857." In the same year Jules Marcou refers to the Jurassic of this region in his *"Lettre sur les Roches du Jura."* This was followed by three papers by Bauerman in 1859 and 1860; and in 1861 Dr. Hector's account of the *"Country Between Lake Superior and the Pacific"* appeared, being the observations made by him as geologist to the Palliser expedition. During the next ten years papers appeared on the fossils of the British Columbia coast by

*Quoted from Whiteaves, "Mesozoic Fossils," Vol. i, part i, p. 95.

Meek (2), Herr, and J. K. Lord, and its glaciology and general physical features by W. P. and T. A. Blake, Whymper, J. D. Whitney, R. Brown, and Sir Matthew Begbie.*

When British Columbia entered Confederation the terms of agreement included an undertaking by the Dominion Government to conduct a systematic geological exploration of the province, and in accordance with this A. R. C. Selwyn, Director of the Geological Survey, and James Richardson were at once dispatched to the Pacific Coast. Selwyn made a preliminary exploring trip across the province in the first season (1871) from the mouth of the Fraser to the Yellowhead Pass; while Richardson devoted his attention to the geology of the coal-basins of eastern Vancouver Island, and afterwards published several reports which dealt mainly with the coal-formations of Vancouver and Queen Charlotte Islands, and are dated from 1874 to 1878.†

In 1877 the name of G. M. Dawson appears at the head of a report on *"Explorations in British Columbia,"* associated with that of Dr. Selwyn. Small of stature, feeble in constitution, and somewhat deformed, as he was, to this man more than any other was to fall the strenuous labors of pioneering a practically unexplored region nearly 400,000 square miles in area, and fitly described as "a sea of mountains." For the next twenty-five years, Dawson's was the leading figure among the geologists of British Columbia. During this time an immense amount of work had been accomplished, a large part of which was necessarily geographic, and included the mapping of rivers, lakes, mountain ranges and passes. The geographic side of the work received large contributions also from the railroad and other surveys which were being rapidly extended along the feasible traffic routes and political boundaries. The results of the field work conducted by Dawson and his assistants were published from year to year in the reports of the Geological Survey, and summarized in papers contributed to various periodicals and learned societies. Of these summaries, Dawson's presidential address, read before the Geological Society of America‡ in 1901, the year of his death, is most complete, and gives a very comprehensive idea of the work accomplished up to that time. A reconnaissance of the whole province had now been made, and some fairly detailed work done in the regions which were economically important. The geographic features were well known in their main outlines, and in considerable detail in the southern part, though there

*See U. S. Geol. Survey, Bull. 127, p. 186.

†For references see Index Vol. i, Geol. Survey Can.

‡Bull. Geol. Soc. Am., Vol. xii, pp. 57-92.

still remain unclimbed peaks and unvisited valleys for the explorers of the future. The areal geology had also assumed definite shape as regards the main outlines—though a vast amount of work yet remains to be done in working out its details among the mountains. The same may be said of the structural geology. We owe our first geological section of the Laramide Range (Rocky Mountains) to Mr. R. G. McConnell, who first came to British Columbia as Dr. Dawson's assistant, and has since contributed notably to its geology by his own researches in the region about Finlay and Omineca Rivers.* The geology of the Yellowhead Pass section was first done in some detail by Mr. J. McEvoy.† The opening of the Klondike in 1897 drew Mr. McConnell away from the work in British Columbia; and during the years from 1897 to 1901, Dr. Dawson's time seems to have been fully occupied by his duties as Director of the Survey, so that further field-work was impossible for him and devolved upon others.

During the period from 1871 to 1901 the Palaeontology of British Columbia received attention from Messrs. Gabb,‡ Meek,§ Sir J. W. Dawson,‖ Billings,** and Whiteaves. The name of T. Sterry Hunt appears as the analyst of some c[illegible] and rocks from Nanaimo in 1872. Apart from the work of the Survey, contributions to the geology of the province had been made by Bauerman, Harrington, Whitney, Branner, C. A. White, Le Conte, Ledoux, Coleman and others.

The sheets geologically explored and mapped by 1901 covered nearly the whole of southern British Columbia as far north as the 51st parallel while to the north of this a fairly complete reconnaisance had been carried out and the interior part of the province had been mapped as far as the latitude of Fort George and beyond.

With the opening of the Klondike a period of commercial and industrial development began in British Columbia, and this has continued since under the impetus of increased immigration, enlarging markets, and general prosperity. The corresponding development of the population and wealth of the province has both demanded and rendered possible more extensive work than was previously necessary. At the same time improved methods, especially in mining geology, glaciology, and physiography have made it desirable that the mining regions should

*Geol. Survey Can., 1896.
†Geol. Survey Can., 1900.
‡Am. Jour. Sci., 3rd Ser., Vol. x.
§U. S. G. S. Ter. Bull., Vol. ii, p. 351, 1876.
‖G. S. C., 1872-3.
**"*Mesozoic Fossils*," Vol. i.

be studied in greater detail and with regard to modern theories of ore genesis, and that the physiographic and glacial work done to the south of the 49th parallel should be continued to the northward. During the last ten years, accordingly, the Topographic Survey has made considerable progress in the production of contoured maps of the southern part of Vancouver Island and of the Selkirk Mountains adjacent to the main line of the Canadian Pacific Railway. The mining regions have received closer attention, and the general geology has been worked out in many instances in smaller sheets and with greater detail than at first. Among the new names that have risen to prominence during this later period are those of Brock, Cairns, Camsell, Clapp, Daly, Le Roy, Leach, Keele, Evans, Wheeler, Walcott*, and Poole. The mining regions so far dealt with include Nanaimo, Texada Island, Atlin, Hedley, Rossland, the Boundary Creek district, the Crow's Nest coal region, the Lardeau region, and the Telkwa River coal-field.

In the matter of stratigraphic subdivision some advances have been made since Dawson's time, including the work of Walcott in the Rockies, of Clapp in Southern Vancouver Island, and of Camsell and Brock in the southern part of the province. Dr. R. A. Daly's work along the international boundary has resulted in the development of the important idea of "stoping" action in the formation of batholiths, and Dr. Walcott's palæontological work in the Rockies near Field has produced quite remarkable results, carrying our knowledge of many of the higher soft-bodied types of invertebrates back as far as the Cambrian, and throwing much fresh light on the appendages of trilobites.

## Problems of the Future.

As to the probable trend of work for the future, it is of course to be expected that each worker will have his own problems which he hopes to solve. The work of a geological survey is rarely complete before it is obsolete,—rendered so by the necessity for more accurate methods and more detailed work, and by the application of new methods of interpretation, or means of research. The first of these causes is a very cogent one in a new and rapidly-developing country, though even in older lands the desire for more thorough information and the means of obtaining it increase somewhat as time advances. In British Columbia the opening up of the northern part of the province, following the construction of the new railways, will shortly call for more detailed research in that region. An especially important factor in this will be the development of new mineral areas, such as the newly-found anthracite coal-field on the headwaters of the Skeena River. Such discoveries

*Dr. Walcott's reputation as a geologist is, of course, of much earlier origin, and was attained by researches elsewhere.

must call for immediate attention on the part of the Survey, and also attract outside investigators.

In the meantime many of even the older parts of the province, such as the interior of the various mountain ranges, where the roughness of the topography and the denseness of the vegetation combine to make access difficult, have still to receive their first accurate study. Such regions, for example, as the interior of Vancouver Island, the Coast Range, and many parts of the Rockies and Selkirks, are still quite unknown.

The second source of new problems is of a more technical nature, and for that reason of greater interest to the investigators themselves, but is less easily understood by the geological laity. An example of new means of research rendering older work obsolete is that of the introduction of microscopic methods in petrography about the year 1885. Since that time the older rock descriptions have been of little practical value. The introduction of new hypotheses in relation to which the facts must be restudied, is hardly less effective in its demand for a re-survey of the field. Not only must the old facts be observed from a different point of view, but new classes of facts, which were formerly either cursorily dealt with or entirely overlooked, now become important, and require close study.

The new problems which may arise from the second class of causes may be grouped under the various departments of geological investigation:—

(a) *Glacial and physiographical.* Dr. Dawson has published excellent accounts of the physiography and glaciology of British Columbia,* in which he describes among other features a number of ancient and later peneplains in the Coast Range and the interior, and base-levels of erosion below these. Much still remains to be done, however, in the correlation of the various ancient base-levels in different parts of the province as, for example, between the interior plateau and the Coast Range, and along the latter from south to north, connecting with work in the State of Washington† and in the Yukon Territory and Alaska. The carrying of such a correlation eastward into the Selkirks, and possibly the Laramide Range, is also a task for the future, which for its completion requires the topographic mapping of at least considerable parts of those regions. The working out of the glacial history of the province in connection with recent continental uplifts is also a matter in which

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*Geol. Soc. Am.; Bull., Vol. xii, and Science, New Ser., Vol. xiii; Roy. Soc. Can., Vol. ix, Sec. 4; Am. Geol., Vol. vi, 1890.

†See U. S. G. S. Prof. Paper 19 by G. O. Smith and Bailey Willis.

much still remains to be done. Dawson began his career as a glacio-fluvialist, and even his latest work may be said to belong to a transition stage of glaciological theory. In the light of more recent developments, therefore, a recasting to some extent may be expected.

(b) *Structural and Stratigraphic.* In these departments the work of Dawson and his contemporaries has no doubt largely determined the main features of the area. The later men have worked out some structural detail and stratigraphic subdivisions, and have extended the old divisions to new fields, and there is no doubt that further research will provide an abundance of structural detail and petrographic information, probably reveal some small areas of horizons now unknown to exist within the province, and furnish, as is even now happening, palæontological materials new to science*. The new stratigraphic movement in the United States must soon be felt here.

(c) *Economic Geology.* The great advance in knowledge which has occurred during the last ten years has rendered possible a class of work previously unknown. Much of the more recent work of the Survey in British Columbia has been along these lines; and in this department the province may be said to be quite fully up to date, though possibly some ground still remains to be covered. With the discovery of new mineral areas new problems must, of course, arise.

In conclusion, it may be said that no doubt many of the most interesting questions that remain to be answered are as yet entirely unknown, or foreshadowed only in the minds of the specialists to whose departments of work they belong, and to whom we must look for their announcement and solution at the appropriate time.

## References.

Complete References to the Geology of British Columbia may be found in the United States Geological Survey Bibliographic Bulletins, especially Nos. 127, 188, 301, 372, and later annual bulletins since 1908.

The work of the Geological Survey of Canada may be further referred to in detail by consulting the Index Volumes I and II published by the Survey, and their Lists of Publications Nos. 689 and 811 G. S. C.

*See Walcott's papers on *Cambrian Geology and Palæontology*, Nos. 1-7, etc., Smithsonian Miscellaneous Collections.

# The Pleistocene Volcanoes of the Coast Range of British Columbia

By Edward M. Burwash, M.A., Ph. D.

Read February 17, 1914.

During the past summer, the writer visited the Mount Garibaldi region, which is situated in the Coast Range of British Columbia, on the east side of the Cheakamous Valley, and about forty miles due north of Vancouver.

The general stratigraphic and physiographic features of the Coast Range may be enumerated as follows:—

1. The range consists of a complex of rocks which include (i) metamorphic sedimentaries and igneous rocks (intrusive and perhaps also extrusive) of palaeozoic age; (ii) the great granite intrusion of the Coast Range batholith (Upper Jurassic) which makes up the main mass of the range; (iii) a set of post-Eocene, probably Miocene andesitic lavas, which lie upon eroded surfaces of the granite and the palaeozoics, overlapping their contact in some places, and (iv) a set of lavas of Pleistocene and post-Pleistocene age.

2. The sculpturing of this mass reveals the fact that it has been subjected to several periods or cycles of erosion during which a fairly constant level was maintained, and erosion progressed, in some cases far, toward peneplanation. These cycles were separated from each other by periods of uplift. The erosion cycles of which evidence remains are as follows:—

(i) A Cretaceous erosion surface on which the basal member of the Eocene deposits of Burrard Inlet may be observed resting. As the intrusion of the granite has been placed in Upper Jurassic times, this may represent the first planation subsequent to that event. The removal of the original batholith roof must therefore have taken place in this interval.

(ii) The surface on which rest the post-Eocene lavas in the Garibaldi region. As these lavas cut the Eocene of Stanley Park these old valley floors on which they are found lying round Garibaldi Lake must have undergone erosion for a considerable time after the surface last described. The contact which represents this surface is observable on the sides of Black Tusk Ridge, and on the south-western side of Lake Garibaldi, where it has been sectioned by later valleys.

(iii) The summit level of the range shows a marked degree of concordance between summits, some of which, like those of White and Black Mountains, near Vancouver, display a considerable flat area. Viewed from Mount Seymour, for example, the summits coalesce to form a remarkably even skyline, which suggests that they are remnants or descendants of a former peneplain. Castle Towers, the Battlements, the Sphinx, and other summits in the Garibaldi region are remnants of this surface.

The relation of this plain to the surfaces on which the Miocene lavas lie (No. ii), has not as yet been very definitely decided. These lavas, so far as known, lie in most cases on surfaces much below the level of adjacent granite summits, and their upper parts themselves form the summits of such mountains as the Black Tusk and "the Table." Whether the flat top of the latter represents the original Miocene surface of the flow is a question of interest which must, in the absence of definite data, remain for the present unanswered. It seems at least a possible hypothesis that the valleys in which the lavas solidified were of early Miocene age and cut in an Eocene peneplain, now represented by the level of the concordant summits.

(iv) Below the upper part of these lavas and in some places over 1,200 feet below their base, there is a well-marked planation represented by the flat top of the spurs between the tributaries of the Cheakamous River and by such level areas as the Black Tusk Meadows. The edge of this level, forms the upper rim of the east side of the main Cheakamous Valley. The plain slopes upward toward the axis of the range at an angle of about 7 or 8 degrees. It is still an open question whether the summit level is merely the continuation of this upward slope. If not, the flat tops of Black Tusk Ridge (below the summit), and of Panorama Ridge are possibly part of the upper summit level of Pliocene age and correspond to the Methow peneplain or summit level of the Cascades in the State of Washington, as described by Willis and Smith.* The Black Tusk Meadow stage is in that event to be correlated with the Entiat stage of the writers just mentioned.

The upper stages of erosion so far mentioned have, as will readily be supposed, been so modified by the later action of extensive ice-sheets, that sharp distinctions between surfaces which are in reality unsympathetic have been lost in the general rounding off.

(v) Below the plain last described, which can be readily followed for a mile or more eastward from its edge, and in parts farther, lie

*U.S.G.S.—Professional Paper 19.

the profound U-shaped valleys of the main fiords and rivers, and their more V-shaped tributaries. A typical example of such a valley is that of the Cheakamous—which is nearly 4000 feet deep on the west side of the Garibaldi area, and after uniting with the Squamish forms the fiord of Howe Sound. These deep valleys have evidence of two distinct stages of formation, an earlier V-shaped gorge, in the lower part of which a U-shaped valley has at a later time been excavated. The resulting profile is as shown in the accompanying diagram. (Fig. 1). In some cases a large outer U-shaped valley seems to contain a distinct inner valley of comparatively shallow depth.*

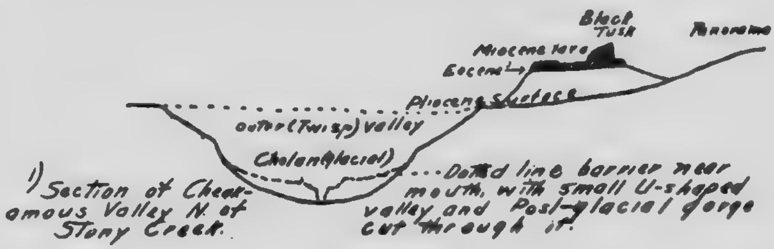



FIG. 1

The date at which these greater canyons were cut has been fixed by Willis and Smith† as probably interglacial for the Cascade region of Central Washington, or possibly pre-glacial. The lines of drainage which they represent are said by LeRoy‡ to have been determined in early Cretaceous or late Jurassic time, but whether this be true or not it is evident that the valleys as they now exist are much later than the Miocene (or at all events post-Eocene) lavas, since the mature erosion of the Black Tusk Meadow stage (Entiat or Methow) was developed after the lavas and before the cutting of the main valleys. From evidence collected near Vancouver, however, it appears that both the periods of glaciation of which we have distinct evidence in the region were later than the excavation of these valleys, that is, there have been two extensions of the glacial ice, with an intervening recession, since the valleys reached their present depth. There is no evidence known to the writer of any earlier ice advances in the region unless some glacial sections to be seen at Departure Bay are to be considered conclusive.§ It would seem more probable that such very large valleys represented

*This stage probably correlates with the Twisp and Chelan stages of Willis and Smith.
†Op. Cit.
‡O. E. Le Roy, Geol. Surv. Can. No. 996.
§Burwash, Contributions to Canadian Biology, 1906-1910, p. 301, published by the Department of Marine and Fisheries, Ottawa.

pre-glacial uplift, and subsequent erosion. This fits well the modern theory which attributes glacial periods to impoverishment of the carbon dioxide of the atmosphere, due to previous uplift and extensive erosion with resulting carbonation of fresh rock-material.

(vi.) There is evidence that during the Pleistocene the sea stood at least six hundred feet higher than now, and perhaps a thousand. The main valleys were then occupied by ice which extended below the sea-level. Where the valleys widened near their mouths the spreading of the ice caused a corresponding decrease in depth and erosion was less intense. The valleys were consequently shallower near their mouths than farther up. When the ice withdrew and the land rose they at first probably contained lakes—dammed by rounded rocky barriers across the lower part, over which the streams cascaded, as the outlet of Lake Buntzen does. They have now cut through these barriers picturesque gorges, of which those of the lower Capilano and Seymour Creeks are good examples.

Co-ordinate with these cuttings* in the bottoms of the major valleys, are the cirques formed by still existing neves and glaciers on the higher slopes of the range. These are often very extensive and have reduced the original surfaces of the summit level to a system of very narrow branching divides which project above the snow along the top of the range. The withdrawal of the ice from the lower valleys to its present position on the slopes above the 5,000-foot level appears to have been a gradual movement, and many evidences of comparatively recent glaciation are to be found outside the present limits of ice-action.

Superimposed upon several of the topographic features already described are the volcanic cones and lava-flows of the Pleistocene period of vulcanism. The large volcanic cones of Garibaldi and Red Mountain stand above the planated surface which extends eastward from the edge of the Cheakamous valley and lies at an elevation of 5,000 feet and upward. This flattish surface has been dissected by the valleys of Stony and Swift Creeks, which are tributary to the Cheakamous. Above it rise the Table, and Black Tusk Mountains, capped by post-Eocene lavas, a fact which fixes the age of this surface as probably Pliocene. The volcanoes can therefore be of no greater age than the Pliocene. The valleys below it are certainly newer, probably late Pliocene or Pleistocene.

Mount Garibaldi, viewed from the north, can be clearly distinguished to be a cone standing above this surface of planation, which passes under it with a gentle slope upwards toward the east. The cone itself is

*Stehekin stage of uplift of Smith and Willis.

some 3,500 feet high, which gives its top a total altitude of about 8,700 feet. It appears to consist, in great part, of loose fragmentary materials with some lava streams interbedded. The materials are of two distinct colours, a brownish red, and a light gray. The cone itself has been much dissected, and the faces remaining are largely those of large cirques, with sharp aretes between. In the cliffs, which form the walls of the cirques, the stratiform structure of the cone may be seen to good advantage. The dip of the layers of deposit away from the centre is especially distinguishable. It is doubtful whether any part of the summit can be recognized as forming part of the crater. The erosion on the east and west sides has been apparently more severe than on the north and south, so that the remains of the cone have now the form of a narrow topped ridge, which extends in a north-and-south direction. Evidence that its activity was subsequent to the formation of the main valleys, or that it remained active after their formation, is found in the fact that some of the lava-streams flowed over the edge of the Cheakamous valley, and may be seen extending down its slopes at points along the Lillooet road. Some blocks of the lava have also been carried by the earlier ice-sheet over the high ridges to the south, and are now to be seen in the lower of the two till sheets, which represent the two periods of ice-advance in the Capilano valley.*

Red Mountain is much smaller than Garibaldi. Its cone in itself has an altitude of about 1,500 feet, and a total height above sea-level of 6,500 feet. It stands on the edge of what was at one time the upper valley of Stony Creek, or the glacier corresponding to it, and is now the valley of Garibaldi Lake. The cone rests partly on the eroded surface of Miocene (?) lavas and partly on granite of the Coast batholith. These underlying rocks take the form of a basin or caldera, in which the volcano stands, and which it nearly fills. The western side of the depression is a cliff of granite, which curves around that side of the cone through a considerable arc. In addition to this, there is, on the north-eastern side of the mountain, a rather remarkable remnant of a large lava-flow. It once flowed down into the valley of Garibaldi Lake, where its lower part seems to have been cut away by the glacier or stream which then occupied the valley. Its upper part is also truncated and presents a rugged cliff, facing the cone. This might also be considered as part of the rim of the caldera. The pinnacle of Miocene lava which rises on the eastern side of the cone, can be accounted for in the same way, as a remaining part of the ring of an ancient caldera.

*See articles by A. T. Dalton, *"Canadian Alpine Journal,"* 1908, p. 205, and by J. Porter, *"Northern Cordilleran,"* 1913, p. 48.

As to the formation of this basin, or caldera, of whose rim three segments remain, three divergent hypotheses were suggested by facts observed in the field: (1) A glacial cirque in which the vent happened to open. (2) A caldera resulting from the giving way of the surface under the cone, and its partial sinking. (3) A caldera due to the destruction by explosion of an earlier cone.

The first of these hypotheses is open to several objections: (a) A salient angle on the side of a deeper valley is an improbable place for the formation of a cirque. (b) The truncated lava-flow above described would have to be accounted for by one of the other two hypotheses in any event, and (c) The glaciated condition of a part of the surrounding Miocene lava which might represent the discharge from the cirque, may also be due to a small glacier which was formed on the side of the cone itself.

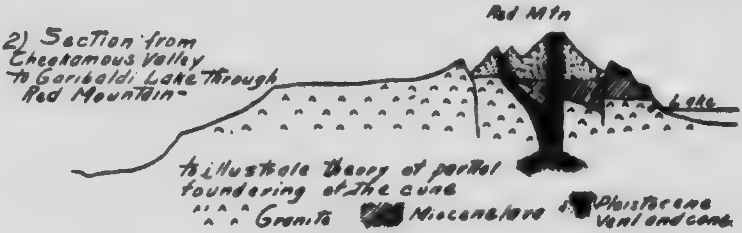



FIG. 2

The second hypothesis has been advanced in the case of the caldera of Mount Mazama in Oregon, but is combated by the mechanical difficulty of causing a fault block to sink into an opening which narrows rapidly downward so far as the scarps exposed would indicate. There is no observed evidence of crush-zones or other criteria of faulting. The fact that the upper part of the truncated lava flow cannot be traced in the present cone would indicate that the foundering, if any, was complete, and the present cone built up subsequently. The accompanying figure would indicate the condition of a partial foundering, in which case flows which extended outside the caldera would be faulted, but probably recognizable on the sunken central part of the cone. (Fig. 3).

The hypothesis of destruction by explosion remains the most probable of the three. It is open to the contention that the fragmentary products of the explosion are not to be found around the vent. This objection may be answered by postulating their removal by glaciation. The surface of the granite outside the caldera on the west, slopes down rather steeply for about 600 feet, to the level of the surrounding country, as if that part of the underlying rocks covered by the volcano had been protected from erosion (see fig. 2), so that the cone stood on a

plinth of a considerable elevation. This would tend to prove that the earlier cone was formed in early Pleistocene, if not pre-Pleistocene times.

Red Mountain, which stands within this basin, consists of a double cone. The eastern and higher summit has a poorly marked crater and has probably suffered a considerable amount of attrition at the top, although its outer slopes show little evidence of alteration by erosion. The materials composing it are largely of the gray coloured lava with which are mixed pieces of the underlying granite and of the Miocene lavas through which the vent was opened. The western cone is about 100 feet lower, and consists of reddish brown lava. It has a fairly complete crater, which was estimated at some 200 feet in depth and about 300 in diameter. The walls of the crater have been breached on the north and south sides

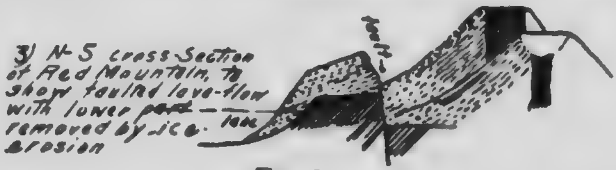



Fig. 8

by the later lava-streams to a depth nearly as great as the present bottom of the crater. The flows from this cone toward the north have filled the valley of Stony Creek for about a mile and a half. The barrier thus formed is the dam behind which the waters of Garibaldi Lake are retained. The outlet of the lake runs along the depression between the edge of the lava flow and the northern wall of the valley for about a mile. It then enters a recess in the edge of the lava where a smaller lake results. The outlet of this lake, at ordinary stages of the water, is subterranean, passing apparently under the lava flow and emerging in the lower valley through the talus at the foot of the cliff known as "the Barrier," which is cut in the lower edge of the lava flows, apparently by the undermining action of the stream at its exit from its subterranean course. The section of lava seen in the cliff shows that there were several flows, the earlier apparently of gray colour and the later brown. The earlier flow may therefore have come from the eastern crater, but as to this no proof exists except the coincidence in colour.

The history of the volcano would then appear to be that in the first stages which occurred early in the Pleistocene, a cone was built up, of which the present eastern cone may have been a part, or the whole. This cone was blown away or sank to its present position owing to the removal of material beneath it at about the close of the glacial period. Subsequently the western cone was built up and flooded the valley to

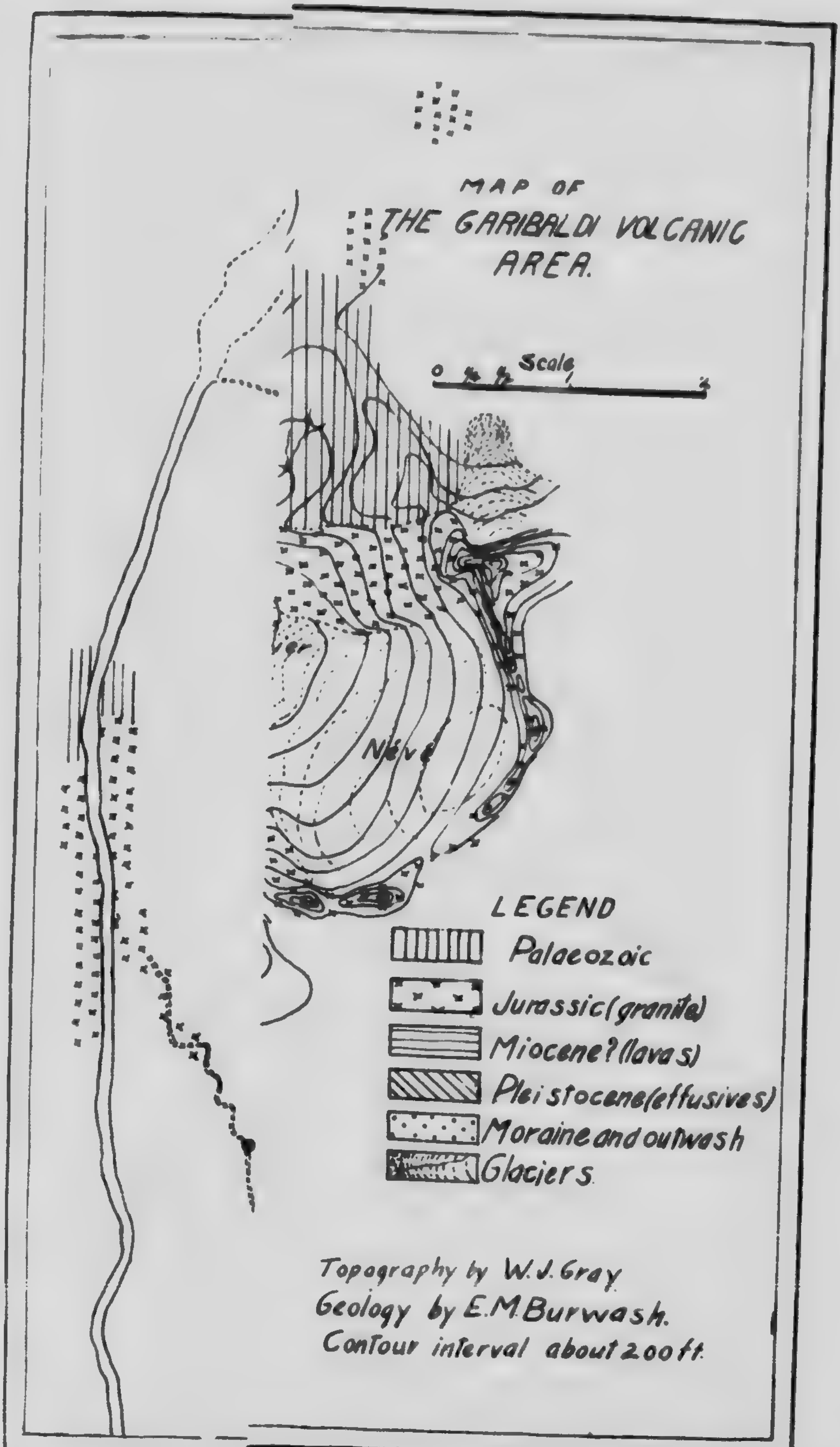
the north of it in the manner already described. Garibaldi Lake, the Barrier, and the western cone, are then all of post-glacial date.

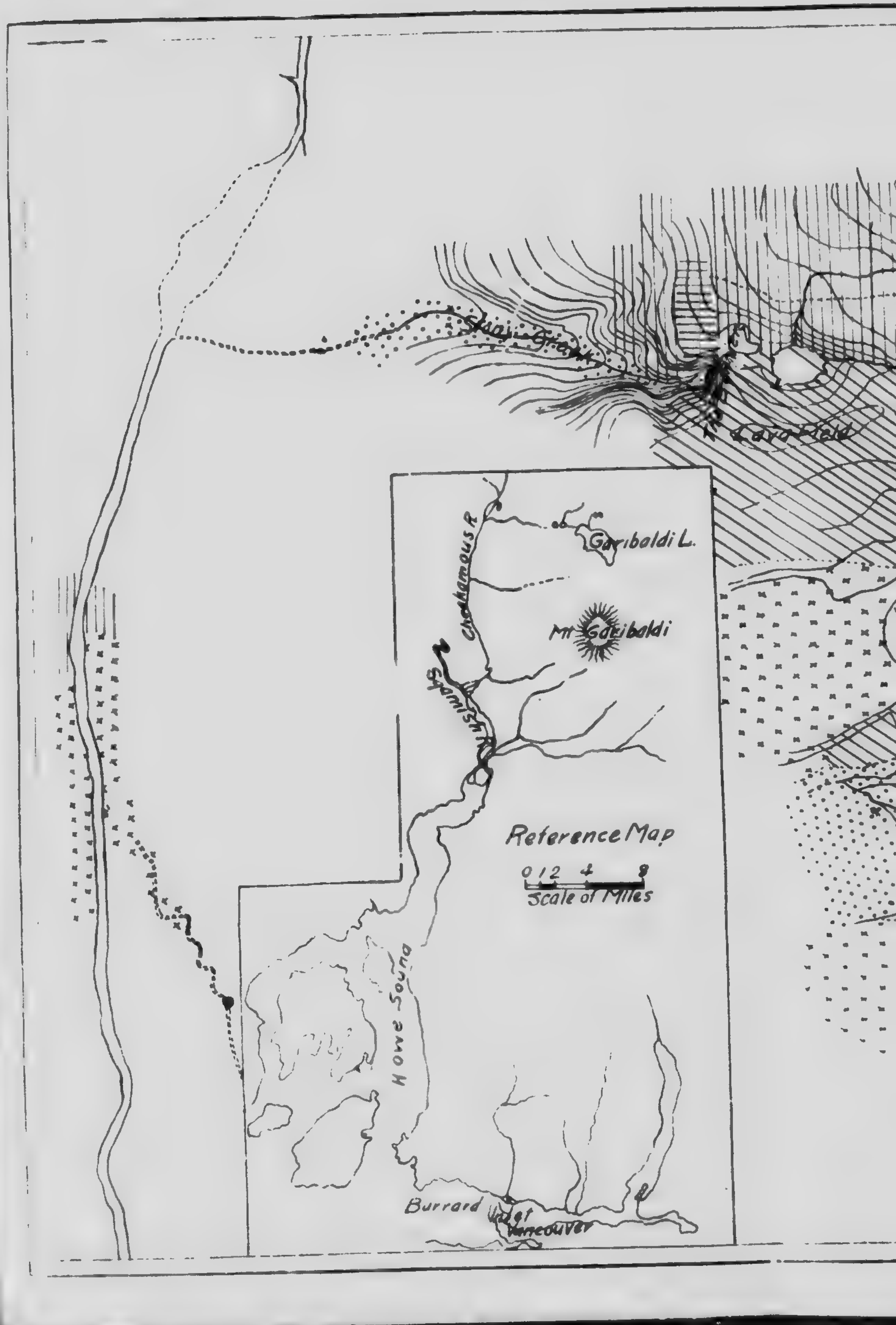
At the eastern corner of Black Tusk Meadows a cascade falls over a low ridge some 200 feet high, and ascending the slope beside it one reaches a hanging valley occupied by a number of small lakes. The valley shows evidence of comparatively recent occupation by a glacier. On its southern side are large snowfields, reaching upward to near the top of Panorama Ridge. The largest of these discharges by two glacier tongues into the valley. The two glaciers—known as the Helmet Glaciers—are separated by an elevation which was at first thought to be a moraine, but on closer examination was found to be a small tufa cone some 500 feet in height. There is a small crater at the top, some 60 feet deep, which was partly filled with water and ice at the time of the writer's visit. The cone has been cut in one place by a stream from the glacier, which abuts against its southern side, and a good section of its faulted layers of tufa can be seen. Helmet Lake, which lies in the valley in front of the tufa cone, has been largely filled by the scoriae carried down by this and other glacial streams.

A small flat topped elevation, several miles to the east of Mount Garibaldi on the east side of Rampart Lake, has every appearance of being another subsidiary vent, but was not visited. Looking northwestward along the range from Black Tusk Mountain, there are several other peaks which, from their form and topographical environments, suggest a similar origin. These remain for future study.

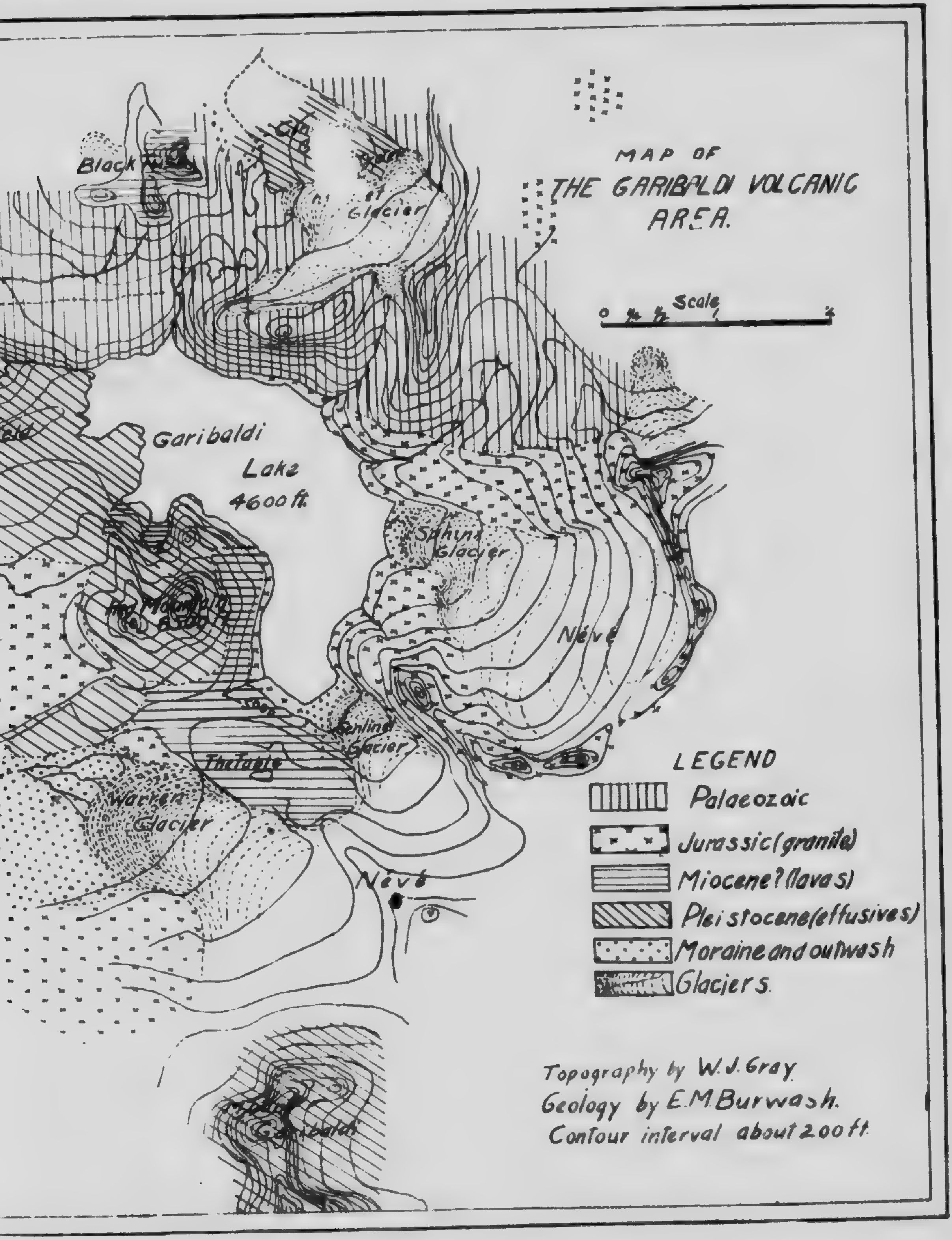
The time at which this volcanic action began, and the period through which it extended, can be fixed with a considerable amount of exactitude. The platforms through which the vents were opened are erosion surfaces considerably later than the Miocene lavas, probably, therefore, Pliocene, and possibly early Pleistocene. The commencement of the vulcanism may, therefore, extend as far back as the Pliocene. The finding of lava erratics of this period of activity, in the lower till sheet of the Capilano valley, proves that the vulcanism was already active during the first period of maximum glaciation, since the till marks the retreat of that ice sheet.

On the other hand the latest lavas from the western cone of Red Mountain, and the cone itself, are not only entirely unglaciated, but the lavas are found over-riding glaciated rocks and moraines, and filling a valley whose U-shape is due to the lesser glaciers of the later part of the ice-period, when local glaciers occupied the present lower valley system, but there was no general movement of a wide-spreading ice sheet or glaciers which spread beyond the limits of the present deeper (Twisp) valleys.


MAP OF
THE GARIBALDI VOLCANIC
AREA.
0 ¼ ½ Scale 1 2
Névé
LEGEND
Palaeozoic
Jurassic (granite)
Miocene? (lavas)
Pleistocene (effusives)
Moraine and outwash
Glaciers
Topography by W.J. Gray
Geology by E.M. Burwash.
Contour interval about 200 ft.



Lava Field
Garibaldi L.
Cheakamous R.
Mt Garibaldi
Squamish R.
Reference Map
0 1 2 4 8
Scale of Miles
Howe Sound
Burrard Inlet
Vancouver


MAP OF
THE GARIBALDI VOLCANIC
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Contour interval about 200 ft.
Garibaldi
Lake
4600 ft.
Black Tusk
Glacier
Sphinx
Glacier
Névé
Sentinel
Glacier
The Table
Warren
Glacier
Névé
Garibaldi

The relation of the successive intrusions to which the Coast Range area has been subjected forms a subject for careful lithological study. The extension of the line of Pleistocene or Recent volcanoes, which girdles the Pacific Ocean, into the geological province represented by the Coast batholith, practically closes one of the few remaining gaps in the "girdle of fire."

The products of the Pleistocene vulcanism of the Coast Range, including lavas and pyroclastics, constitute a formation equivalent to rocks of a similar age and type in the United States. It is proposed that the name Garibaldi Group should be applied to them for the locality here described. The term would serve to distinguish them, not only from formations of a different age, but also from Pleistocene deposits of glacial and aqueous origin, with which the volcanics are to a large extent, though not entirely, contemporaneous.

# Entomology in British Columbia

A REVIEW BY R. C. TREHERNE, B.S.A.

(Read April, 1913)

The study of Entomology in British Columbia is still so young that past records are fresh in our minds. All records point to one man, the Rev. George W. Taylor, F.R.S.C., F.Z.S., as the first active entomologist in this Province. He settled on Vancouver Island about the year 1877 and studied for the ministry. In 1881 he was made a Fellow of the Royal Society of Canada, obtaining this honour largely through his interest in entomology. During the next few years Mr. Taylor proved himself an active collector and a keen student of entomology. At the Annual Meeting of the Entomological Society of Ontario, held in London, Ont., October 15, 1884, he presented the Society with a collection of Diurnal Lepidoptera, through the medium of Mr. James Fletcher.*

In 1885 a paper was published on the "Entomology of Vancouver Island; Notes on 76 species of Cicindelidæ and Carabidæ Collected Near Victoria, Vancouver Island" by George W. Taylor, Victoria, B. C.† In this paper Mr. Taylor says:—"The beetles enumerated below were all taken by myself in the neighbourhood of Victoria, Vancouver Island, during the past few seasons. . . . . A large number, (40) of those I now record, are new to the Canadian fauna, that is, as far as my knowledge of the same goes, and some of these additions are very interesting ones." Mr. Taylor adds further on in the same paper, "I have added to the list two species taken on the mainland of B. C. by Mr. James Fletcher (of Ottawa), in 1883, and very generously given to me."

A demand for entomological knowledge was evidently beginning to be felt within the Province at this time, for in 1887 Mr. Taylor was appointed Honorary Provincial Entomologist for British Columbia.

In 1893 a "Report on the Entomology of British Columbia" was prepared by Messrs. W. H. Danby and C. De Blois Green.‡ In the text of this report we receive light on the status of entomology at the time. A preliminary check list of Lepidoptera collected in B. C. is included therein, and the following note is recorded: "The names of species belonging to the Geometrina cannot be included in this report for the reason that, to get them named by competent authorities takes

*Proceedings B. C. Ent. Soc. No. 2, N. S. 1912, page 2.
†Canadian Entomologist, 1885.
‡Bulletin of the Natural History Society of B. C., 1893, Art. III.

considerable time, in consequence of the great care necessary to avoid mistakes." Other orders are also mentioned. The report says, "No great work was done (in 1892) in collecting Coleoptera, with perhaps the exception of capturing that rare beetle *Ulochaetes leoninus* which is one of the few short-winged species of this family in our fauna, a single species being taken at Alert Bay* and a few other rare specimens being collected in Victoria, names of which have yet to be received". . . . . "Many Diptera were collected, names unknown, with the exception of *Anisopogon ludius,* n.sp., named by Mr. D. W. Coquillet, Los Angeles, Calif., which was captured at Goldstream on Mt. Austin."† . . . . "Amongst the Arachnidæ very little was achieved, the species taken being collected more for friends than study. However, a few specimens were kindly named by Dr. Nathan Banks, of Washington, D. C., amongst them being a new species of *Pardora and Coriaractine,* also a variety of *Epeira insularis,* Hentz."

The names of men engaged in the study of entomology about whom we have printed records increase as years progress. In 1901 we read of the formation of a British Columbia Entomological Society, of which the Rev. George W. Taylor naturally became the first President. The following account records the initial steps and the subsequent occurrences relative to this British Columbia Entomological Society, in the words of Mr. Tom Wilson, Vice-president of the Society:—"In 1901 the late Dr. Fletcher had occasion to visit British Columbia in connection with the work of his Department. During his stay in Vancouver I had the opportunity of introducing him to Mr. R. V. Harvey of the city, who was then Principal of Queen's School. In discussing matters in connection with the entomological work in B. C., the idea was suggested to form a Society whose special object was to unify the work of those particularly interested in the study of insects in the Province. The idea came to maturity and the Society was formed. For two years we had a live Society here in Vancouver with such members as Messrs. G. W. Taylor, Sherman, Bush, Dashwood-Jones, Ed. Wilson, Draper, Marrion, Bryant, L. D. Taylor, Hanham and myself in more or less regular attendance. . . . . Regular meetings were held in Vancouver and reports were issued semi-monthly on matters of interest. These reports took the form of letters, which were forwarded from one member to the other by the members themselves."‡

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*Canadian Entomologist, XXIII., page 283.
†Canadian Entomologist, XXV., page 21.
‡Proceedings of the B. C. Ent. Soc., No. 1, N. S., 1911.

The first meeting of this Society was called on March 13th, 1902, in Queen's School, Vancouver. Biennial meetings appear to have been held from this date until February 6th, 1905. The records of meetings are preserved in a minute book, no attempt being made to print them for permanent distribution and record. Great credit is due Mr. R. V. Harvey, M.A., for his earnest endeavours in maintaining and recording the minutes and transactions of the Society. These are being retained on the shelves of the Society today; and as they contain many notices of important captures and resolutions, they will long be referred to by students of entomology.

On March 29th, 1905, a circular letter was sent out by the Secretary, Mr. Harvey, duly authorized by the Society, stating that the "B. C. Entomological Society has been duly affiliated as a Branch of the Entomological Society of Ontario." The letter further states to the members that "in the future your subscription of One Dollar per annum will cover all the privileges attaching to both Societies, namely, the receipt of the "Canadian Entomologist" monthly and the monthly list of records circulated among the local members."*

Conjointly with the preceding occurrences several important publications were issued, chiefly by certain investigators from the United States, relative to British Columbia Entomological fauna.

Dr. Harrison G. Dyar's List of North American Lepidoptera was notably important and very useful to our local entomologists, who were one and all at this time chiefly concerned with the study of the Lepidoptera. Dr. Dyar's valuable work therefore gave a great impetus to the systematic recording of Moths and Butterflies in the Province of British Columbia.† His book was in general use in the Province in 1903.

In 1904 Mr. August Busch of the U. S. Department of Agriculture published an account of the "Tineid Moths from British Columbia, with Descriptions of New Species."‡ In the preface to this work the author honours, among others, Messrs. J. W. Cockle of Kaslo, B. C., and Theodore Bryant of Wellington, B. C., for assistance rendered in the work.

In 1904 Dr. Dyar again published a valuable work on the "Lepidoptera of the Kootenai District of British Columbia."§ In this work again the experience of Mr. J. W. Cockle is often referred to.

*Minutes of the B. C. Entomological Society, 1905.

†Bulletin No. 52, Smithsonian Institution, United States National Museum, Washington, 1902.

‡Smithsonian Institution, U. S. National Museum.

§Smithsonian Institution, U. S. National Museum.

In 1904, again, largely resultant from the impetus given the science of entomology in the province by Dr. Dyar's List of Lepidoptera, and founded directly upon that list, Mr. Francis Kermode, Curator of the Provincial Museum, Victoria, assisted by Mr. E. M. Anderson, also of the Museum, published a "Catalogue of British Columbia Lepidoptera" from cabinet specimens in the Museum. This list was supplemented and enlarged in 1906 by the officers and other members of the British Columbia Entomological Society, and published by the authority of the Legislative Assembly from the Provincial Department of Agriculture.*

Writing in a letter of November 15th, 1905, from the Entomological Laboratory of Cornell University, Ithaca, N. Y., Mr. J. Chester Bradley describes the trip taken through the Selkirks of B. C. by himself during the summer of 1905. He mentions the fact that he would be glad to determine hymenopterous material, especially specimens belonging to the Evaniidæ and Siricoidea for the members of the B. C. Entomological Society. (No printed record is available for reference).

In March, 1906, the first Quarterly Bulletin of the British Columbia Entomological Society was published. In the opening paragraphs we find the following words:—"Ever since our Society was formed we have laboured under a difficulty which has severely handicapped our work, namely, the distance which separates individual members . . . Attempts have been made to meet this difficulty by sending round MS. notes, but here the neglect of one member has thrown the whole system out of gear. . . . . Last December the Secretary approached the Provincial Department of Agriculture to obtain aid in printing a regular bulletin of our proceedings and work. This assistance has been promised for one year, and it rests with us to prove to the Department the value of our work. . . . . The Bulletin will be published in March, June, September and December, and a copy will be sent to each member."

Some useful notes are given in the opening number of this "Quarterly Bulletin" on records of B. C. Entomological Fauna in Journals to date, and use of these records has been made in compiling lists of B. C. insects in the bulletins that follow.

These quarterly bulletins, 10 in number, continued to be issued until June, 1908, when, in the final number, the last record of an annual meeting of the Society, which took place at Duncans at the residence of Mr. G. O. Day, on Thursday, April 16th, 1908, is found.

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*Check List of British Columbia Lepidoptera, 1906.

Thus for so long—from March 13th, 1902 until April 16th, 1908,—the Society was active and strong; but after this date no further regular meetings appear to have been held until the Society was resuscitated on December 9th, 1911. In the words of Mr. Tom Wilson, Vice-president of the Society:—"Interest gradually waned—not, let it be understood, in entomological work, but rather as a Society."

In these quarterly bulletins between March, 1906, and June, 1908, many records of interest are found, notable among which are the following, all in reference to British Columbia:—"The Coccinellidæ of British Columbia."† "The Buprestidæ of British Columbia."‡ "The dipterous families, Bombyliidæ and Therevidæ."§ "The British Columbia List of Odonata,"** and records in the dipterous family, "Tabanidæ."†† In Bulletin No. 4, December, 1906, records are made in the dipterous family "Tachinidæ"; also references are made to specimens taken in various families in the "Neuroptera." A few captures are also recorded in the coleopterous families, "Cicindelidæ and Cerambycidæ," and in the dipterous family "Asilidæ."‡‡

An extended list of British Columbia "Syrphidæ" is also found§§ compiled from Raymond C. Osburn's paper on "B. C. Syrphidæ, New Species and Additions to the List,"*** while throughout the various issues of bulletins continuous records of Lepidoptera are made.

Messrs. R. V. Harvey, R. S. Sherman, A. W. Hanham, J. W. Cockle, G. O Day and a few others were notable B. C. entomologists for their assistance given to systematic recorders by their collections and field notes in season. The whole series of notes contained in these quarterly bulletins represent good ground work for future students in the province.

On May 27th, 1908, Dr. C. J. S. Bethune in presenting a "Bibliography of Canadian Entomology for the Year 1907," in addition to referring to the records in the various bulletins of the B. C. Entomological Society, reviews the list of Coleoptera collected by Mr. J. M. Macoun in British Columbia, including several species not previously recorded from Canada.††† In the same review Dr. Bethune refers to

†Bull. No. 1, B. C. Ent. Soc., March, 1906.
‡Bull. No. 2, B. C. Ent. Soc., June, 1906.
§Bull. No. 2, B. C. Ent. Soc., June, 1906.
**Bulls. Nos. 3, 4, B. C. Ent. Soc., Sept., 1906.
††Bulls. Nos. 3, 4, B. C. Ent. Soc., Sept., 1906.
‡‡Bulls. Nos. 4, 5, B. C. Ent. Soc., Dec., 1906, March, 1907.
§§Bull. No. 8, B. C. Ent. Soc., Dec., 1907.
***Canadian Entomologist, Vo. XL., No. 1, Jan., 1908.
†††Ottawa Naturalist, XXI., Nov., 1907.

three "New Species of Noctuidæ for 1907" from B. C., recorded by John B. Smith*, and "Notes on the Brephidæ," also by John B. Smith, containing records of original descriptions and new species from British Columbia.†

In addition to all the foregoing lists, records and reviews, the Canadian Entomological Record, compiled and edited from time to time by the Dominion Entomologist assisted by his staff and the leading entomologists throughout Canada, contains many names of captures and records for the Province of British Columbia.

As we pause at this point and glance back over the foregoing resume of British Columbian entomology, it is observed that our entomologists have been chiefly concerned in the study of the Lepidoptera. Other orders in the main have been neglected, and such records as we possess in these other orders have been obtained largely from outside sources. Consequently a large field of entomological investigation and research still lies open in almost every direction.

Having referred thus far only to the systematic side of the study of entomology in British Columbia,—the collecting, mounting and recording of insects,—we find that the economic or applied phase of the science was not altogether neglected; for records of this branch of the subject occur conjointly with the foregoing notes.

No praise is too great for those whose energies have made this province an example to all of the practical value of entomology, which other provinces and states failed to discover until too late. Our work in the future will be to retain this standard of immunity, whatever the cost in labour and expense. We must endeavour at the same time to supplement our knowledge of those insects of economic importance in our midst and indigenous to the province.

In May, 1912, Dr. C. Gordon Hewitt, Dominion Entomologist, established a Field Station for investigational research in entomology, the object being to study the life history and habits of insects in the province, preferably those of economic importance and their control. The writer has the honour, at the moment, of occupying the position of Field Officer for British Columbia under the direction of the Dominion Entomologist. Through the courtesy of Mr. Arthur Brealey, fruit grower, Hatzic, in the Lower Fraser Valley, the Dominion Division of Entomology was permitted to establish temporary field quarters on his farm for the purpose of investigating the life history and habits of Otiorhynchus ovatus, the Strawberry Root Weevil, an insect which was

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*Trans. Am. Ent. Soc., Philada., XXXIII., 125—143, May, 1907.
†Can. Ent., XXXIX., 369—371, Nov., 1907.

causing very considerable annoyance to the growers in the locality. These temporary quarters at Hatzic were exchanged for permanent ones on the Dominion Experimental Farm, Agassiz, also in the Lower Fraser Valley, in the spring of the next year (1913).

In the spring of 1912, again, the Provincial Department of Agriculture appointed Mr. W. H. Brittain, at the request of the British Columbia Board of Horticulture, to the position of Entomologist and Plant Pathologist for the province. Mr. Brittain commenced to work, collecting data on fungous diseases, insects, and the general conditions in the province in April, 1912.

On August 22nd, 1912, the "Father of B. C. Entomology," the Rev. G. W. Taylor, died of paralysis at Departure Bay, Nanaimo.

Under the date of November 12th, 1912, Seymour Hadwen, D.V. Sci., of the Dominion Veterinary Department, Experimental Farm, Agassiz, published an account of the "Economic Aspect and Contributions on the Biology of Warble Flies" under the auspices of the Dominion Department of Agriculture, Health of Animals Branch.

The British Columbia Entomological Society, which had held its last meeting in April, 1908, was resuscitated again at a meeting held in Aberdeen School on December 9th, 1911, with 17 members in attendance at the meeting, and 24 on the membership roll, including the above. A short bulletin was published of the proceedings.

The Society again met in Victoria on January 9th, 1913, being the ninth active annual meeting of the Society, but the twelfth annual meeting from the date of its inception in March, 1902 (allowing thus for the years of dormancy). The proceedings of this last meeting in January, 1913, appeared in print in regular bulletin form in the spring of 1913.

Proper representations were made to the Provincial Department of Agriculture, with the result that a grant of $250 was placed to the credit of the Society in the spring of 1913.

The present status of entomology in British Columbia is bright indeed, and all bids fair for the future. The number of workers is increasing, and the work as it stands today is only at its commencement. The demand for workers is great, and the demand for knowledge is greater. There are few countries with such an interesting entomological fauna. There are few studies that offer such scope for individual research as this same entomological fauna. The field lies open for systematic or economic investigation, for popular or scientific research, for those who may wish to apply their energies towards the furtherance of our knowledge of British Columbian insects and their characteristics.